

DYNAMIC ANALYSIS AND PARAMETRIC
IDENTIFICATION FOR FLEXIBLE
SERIAL MANIPULATORS

BY

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Abstract of Dissertation Presented to the Graduate School
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Vibrations in the small are low magnitude, oscillatory deformations about the mean motion equilibrium as distinct from the deformations in the large which result from external loads and large but slowly varying inertia loads. Vibrations in the small are caused by transient phenomena such as system shocks introduced when starting or stopping a machine such as a manipulator arm. The lowest natural frequency of vibration is one of the best indicators of the overall structural integrity of the manipulator system and is an indicator of the maximum acceptable speed for performing precise arm motions.

This dissertation presents a new formulation for the dynamic model of flexible manipulators. A lumped-parameter arm model is used for tractability. This allows the dynamic equations of motion to be written as a set of coupled ordinary differential equations instead of partial

differential equations. For many manipulator designs, most of the deformation energy appears in the actuator and drive train components. Therefore, an initial model considers flexibility at the joints in the direction of relative joint motion only. The natural frequencies and their corresponding natural modes are determined from an eigenvalue problem obtained from the equations of motion. This formulation is then expanded to include other system flexibilities by introducing pseudo-inputs to represent their motion.

Based on analytical modal analysis a solution algorithm is presented to estimate the dynamic response of the arm due to deterministic exciting forces. This enables the designer to obtain the response of the arm in frequency domain by considering a harmonic exciting force applied to the system.

Although the inertia loads due to the gross motion of the arm have small effect on the natural frequency of the system, they cause the system to deform due to the existence of flexible components. Therefore, the mathematical formulation is extended to include both small and gross motion of the manipulator.

Finally, an experimental procedure is introduced to determine the mass and stiffness parameters of the manipulator in order to represent a true model of the actual system.

CHAPTER I

INTRODUCTION

The combination of computer control and sophisticated multi-degree of freedom mechanical systems has evolved into a new discipline referred to as robotics. The potential for this discipline to greatly increase production and improve its quality has brought our social structure to the verge of a modern industrial revolution. Achieving this goal demands enormous effort to develop robot technology.

Although manipulator arms have been employed in a wide range of industrial applications, exploitation of these mechanisms to their maximum capacity is yet to come. The ability of existing manipulator arms is limited because of technological barriers in (a) "real time" computational control, and (b) design procedures [1].* In parallel with the need for "real time" computations, mathematical formulations are necessary to incorporate complex geometries and dynamic characteristics in the design process. Current manipulator designs maintain

* The numbers in brackets refer to references at the end of this document.

simple geometries and are developed based on trial and error procedure, which is both time consuming and costly. Alternative geometries are much needed to extend manipulator capability for performing more sophisticated tasks, such as obstacle avoidance en route to a destination. In addition, system dynamic properties must be considered in the design procedure to improve the manipulator's operational capacity and precision.

An issue of extreme importance in many manipulator tasks is the precise positioning of the arm's end-effector during movement. As speed of operation increases and the manipulator stiffness decreases, due to a need to decrease the mass of the moving parts, the flexibility of the arm generates inaccuracy in the arm motion. There are two distinct types of structural deformations which cause the arm to deflect about its equilibrium position. These are referred to as deformations in the large, and vibrations in the small [1]. Deformations in the large are non-oscillatory deflections, which are caused by external loads and inertia loads due to the gross motion of the arm. In operation, the links of the manipulator must be constantly accelerated and decelerated, leading to large inertia loads and corresponding deformations. Superimposed on this gross motion and deformation will be small scale oscillations about the equilibrium. These are the vibrations in the small. This dissertation presents a

development of mathematical formulation for analysis of manipulator dynamic behavior, considering its flexible components.

In general, structural integrity is a primary concern in manipulator design. The most direct way to reduce structural deformations is to increase system vibrational frequencies; specifically the fundamental frequency (lowest natural frequency), which is an excellent indicator of the manipulator's structural integrity. Chapter II formulates the vibratory motion of the arm in its stationary configurations from which the natural frequencies and corresponding mode shapes are deduced. In addition, based on Rayleigh's principle, the fundamental frequency is described explicitly in terms of local parameters to demonstrate the effect of each parameter on the magnitude of fundamental frequency. Not only is this frequency an indicator of system rigidity, it also provides a basis to classify the mechanism's operating speed. A manipulator's gross motion is described by a parameter, ω , which indicates the rate of moving a load from one position to another and back. This parameter itself does not indicate whether a system is high speed or not, since what is considered as high speed for a particular mechanism may represent a low speed for another system. Yet, the comparison of ω with the system's fundamental frequency provides a criterion for dynamic

classification of mechanisms. This criterion for single degree of freedom systems has been discussed in detail by D. Tesar and G. K. Matthew [2].

Although the source and magnitude of excitation phenomena are extremely difficult to accurately quantify in general, the system response due to deterministic excitations, such as impulsive forces, offers valuable information about system reaction to dynamic shocks in the motion of manipulator arms. A linear formulation is presented in Chapter III to analyze system dynamic response under applied exciting loads. Also, the effect of nonlinearities, such as Coriolis and centripetal accelerations on the system response, is discussed in this chapter.

As mentioned previously, increasing the speed of a manipulator generates larger inertia forces which cause the arm to deform about its equilibrium position. To predict arm deflection due to inertia loads, the equations of motion are derived in Chapter IV, considering both gross motion and arm deformation, along with the coupling between the two motions. Ultimately, these deformations may be compensated for by modifying the nominal actuator commands.

Accurate analysis of flexible arm behavior requires precise knowledge of system mass and stiffness properties. These parameters are commonly determined from the

continuously distributed model of the link components, or finite element methods using the mechanical properties of system elements. Manipulator link components, in general, are not uniform beams; therefore, the continuous model falls short of being practical. This problem is overcome by dividing the link elements into many small segments using finite element methods; however, this approach increases the system's representative degrees of freedom, leading to complication in analysis and numerical computations. An experimental procedure is proposed in Chapter V to obtain system mass and stiffness parameters based on the modal analysis concept. This approach offers an accurate representation of the arm since the dynamic parameters are identified so that the model behavior will match those of the actual system. Furthermore, these parameters are obtained from the prototype mechanism in its assembled form. Therefore, the effects of joint link connections on the component flexibility are included in the identification process.

In order to illustrate the application of these formulations, Chapter VI contains analysis of an example involving a flexible manipulator arm.

During the past few decades, considerable attention has been devoted to the dynamics of high-speed flexible mechanical systems. Two remarkably complete literature surveys [3,4] present a review of research conducted on

this subject prior to 1972. Imam, Sandor, and Kramer [5] present a deflection analysis of planar linkages, where the system mass and stiffness matrices are constructed, utilizing the permutation vector method of structural analysis, and frequent recalculation of eigenvalues is avoided by using the "Rate of Change of Eigenvalues Method." Several publications [6-10] utilized the lumped-parameter approach for dynamic analysis of flexible planar linkages. Kohli and Sandor [11] extended this approach to investigate dynamic behavior of spatial mechanisms with elastic components.

The coupled, nonlinear, partial differential equations of motion for a slider crank were developed by Chu and Pan [12] to include the transverse and longitudinal flexibility of the coupler link. In 1978, Sanders and Tesar [13], based on their analytical and experimental research, argued that deflection of mechanisms with elastic members can be predicted by a quasi-static method (higher time derivatives of small motion are considered to be negligible) if the mechanism operating speed is less than five percent of the system fundamental frequency. Hains [14], through an independent investigation, confirmed the conclusions drawn by Sanders and Tesar.

An analytical and experimental investigation was conducted [15] to study the dynamic behavior of a

high-speed planar four bar with elastic members. The equations of motion are derived using the assumed mode method by modeling the four bar as a cantilever beam (crank), connected with two simply supported beams representing the coupler and follower links. Dubowsky and Maatuk [16] present a formulation for the dynamic equations of motion for flexible manipulators. The manipulator's links are modeled as a series of point-connected, slender beams, using Euler beam theory. System equations of motion are then derived by application of Lagrange's formula. As mentioned previously, this formulation is limited to mechanisms with uniform link components. Furthermore, the significance of system natural frequencies in the analysis of flexible mechanisms has not been addressed.

The transfer matrix method is utilized [17] to develop a formulation for the analysis of flexible planar manipulators. The dynamic behavior of system components is described by constructing matrix elements which relate the motion and internal forces at one end of each component to those of the other end. The system transfer matrix is then built up while traversing the sequence of attached Bernoulli-Euler beams and bodies. The manipulator's natural frequencies and frequency response are obtained from the system transfer matrix and mechanism boundary conditions. This method neglects Coriolis and

centripetal accelerations for determination of system response. The transfer matrix approach is extended to spatial manipulators in [18] using a lumped mass model. By this procedure a CRL master-slave manipulator is investigated. The effects of configuration and payload on frequency response are studied both analytically and experimentally.

Tesar and Thomas [19] developed a lumped parameter model for planar three degree of freedom manipulators to determine system natural frequencies. Equations of motion are derived in terms of absolute deflection of mass elements using a Newtonian formulation.

An analytical method is presented [20,21] for the dynamics of spatial mechanisms containing complex-shaped, flexible links. The mass and stiffness properties of links are represented by a set of matrices deduced from finite element methods. These parameters are defined in the moving coordinate systems attached to the links. Therefore, they are invariant with respect to variation of the mechanism's configuration. Equations of motion are derived using Lagrange's formula and the 4×4 matrix dynamic analysis technique. System motion is defined in terms of absolute perturbed motion of finite element nodal points. This paper provides a significant and beneficial analytical formulation for dynamics of flexible mechanisms; however, it is not a practical procedure for

considering system flexibility in the design process. Although component mode synthesis coordinate reduction is introduced to decrease the system's degree of freedom, numerical integrations are cumbersome.

This document presents a lumped parameter model for dynamic analysis of flexible serial manipulators. The equations of motion are defined in terms of relative perturbed motion of mass components using Lagrange's formulation. Not only is this formulation relatively efficient for numerical integration, it also provides an explicit representation of system fundamental frequency in terms of local mass and stiffness parameters. Therefore, the effect of component flexibility and inertia on the system's vibration can easily be investigated through this formulation.

CHAPTER II

DYNAMIC MODEL OF FLEXIBLE MANIPULATOR ARMS

The main objective of this dissertation is to formulate the vibration behavior of manipulators, therefore providing a basis to include the system's flexibilities in the design procedure. Precision of the manipulator's operation must be assured in order to acquire a high quality product. Thus, it is required to design the system so that vibratory motions are diminished under operating conditions. The most direct way to achieve this goal is to keep the frequency content of exciting phenomena, produced by the system's operational function, far below the system's fundamental frequency. Therefore, the fundamental frequency can be utilized as a design criteria to increase system rigidity, leading to precision at higher operational speeds.

This chapter presents a mathematical model for dynamic analysis of flexible serial manipulator arms. Of particular interest is computing the mechanism's natural frequencies, specifically the lowest one (fundamental frequency), and the corresponding mode shapes. This formulation is simple to implement on digital computers

and allows the designer to consider only the dominant, flexible components; enabling him to select the simplest, yet most accurate, representation of the mechanism to eliminate unnecessary computations. The present work is based on the formulation developed by Thomas and Tesar [22], for dynamic analysis of manipulators with rigid elements. The kinematic representation of serial manipulators developed in [22], is briefly reviewed here.

2.1 Kinematic Representation of Serial Manipulators

A serial manipulator is an assembly of links and revolute or prismatic joints (most of the lower kinematic pairs are a combination of these joints), as shown in Fig. 2.1. The motion of each joint is defined by a parameter, ϕ_n , representing a relative angular displacement, θ_n , about the joint axis, $\underline{S}_n$, (revolute joint), or a relative axial motion, S_n , along $\underline{S}_n$ axis (prismatic joint). Two successive joint axes, $\underline{S}_j$ and $\underline{S}_k$, are connected by a link member along their common perpendicular vector, $\underline{a}_{jk}$. Parameters, a_{jk} and α_{jk} , which are the distance along $\underline{a}_{jk}$, and the angle of twist about $\underline{a}_{jk}$ between $\underline{S}_j$ and $\underline{S}_k$, define the link elements, as shown in Fig. 2.1. To describe the manipulator's motion, a global Cartesian coordinate system, along with a set of local coordinate frames, are

are chosen. The global system is a fixed frame in space located at the first joint with axes X^* , Y^* , and Z^* , where Z^* is selected along joint axis, $\underline{S}_1$. The local frames $(x^{(j)}, y^{(j)}, z^{(j)})$, are a set of Cartesian reference coordinates fixed to the link elements with $x^{(j)}$ lying along $\underline{a}_{jk}$ and $z^{(j)}$ along $\underline{S}_j$. Thus, the direction cosines of $\underline{a}_{jk}$ and $\underline{S}_j$ in terms of the j th coordinate system, are $\underline{a}_{jk} (1,0,0)$ and $\underline{S}_j (0,0,1)$.

A body in space is completely defined by its orientation and the position of a point on the body. The orientation of link jk is described by the direction cosines of $x^{(j)}$, $y^{(j)}$, and $z^{(j)}$ in terms of the global frame. The direction cosines of an arbitrary vector $\underline{Q} (X_q^*, Y_q^*, Z_q^*)$ can be obtained by a rotational transfer of its direction cosines $\underline{Q}^{(j)} (X_q^{(j)}, Y_q^{(j)}, Z_q^{(j)})$ defined in the j th local reference frame. Let T_j designate a 3×3 rotation matrix which relates the j th frame to a coordinate system with common origin to the j th frame and with axes parallel to the global coordinates. We can write

$$\underline{Q} = T_j \underline{Q}^{(j)} \quad (2.1)$$

Therefore, the direction cosines of $\underline{a}_{jk}$ (X_{jk}^* , Y_{jk}^* , Z_{jk}^*) and $\underline{s}_j$ (X_j^* , Y_j^* , Z_j^*) in terms of the fixed reference frame are

$$\underline{a}_{jk} = T_j \underline{a}_{jk}^{(j)} \quad (2.2)$$

$$\underline{s}_j = T_j \underline{s}_j^{(j)} \quad (2.3)$$

Substituting the values of $\underline{a}_{jk}^{(j)}$ and $\underline{s}_j^{(j)}$ into Eqn. (2.2) and (2.3),

$$\underline{a}_{jk} = T_j \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2.4)$$

$$\underline{s}_j = T_j \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (2.5)$$

Equations (2.4) and (2.5) imply that the first and third columns of T_j must be formed of vectors, $\underline{a}_{jk}$ and $\underline{s}_j$, respectively. Furthermore, $y^{(j)}$, in a right hand coordinate system, is formed by a vector crossproduct of $z^{(j)}$ and $x^{(j)}$

$$y^{(j)} = z^{(j)} \times x^{(j)} = \underline{s}_j \times \underline{a}_{jk} \quad (2.6)$$

The vector crossproduct, $\underline{S}_j \times \underline{a}_{jk}$, forms the second column of matrix T_j , since direction cosines of $y^{(j)}$ in terms of the j th local reference frame are $(0,1,0)$. The rotation matrix T_j can then be represented as

$$T_j = [\underline{a}_{jk}, \underline{S}_j \times \underline{a}_{jk}, \underline{S}_j] \quad (2.7)$$

The direction cosines of $\underline{S}_j$ and $\underline{a}_{jk}$ can be obtained recursively from the initial direction cosines

$$\underline{S}_1 (X_1^*, Y_1^*, Z_1^*) = (0,0,1) \quad (2.8)$$

$$\underline{a}_{12} (X_{12}^*, Y_{12}^*, Z_{12}^*) = (C\theta_1, S\theta_1, 0) \quad (2.9)$$

and the following expressions

$$\underline{S}_j = T_{j-1} \begin{bmatrix} 0 \\ -S\alpha_{(j-1)j} \\ C\alpha_{(j-1)j} \end{bmatrix} \quad (2.10)$$

$$\underline{a}_{jk} = T_{j-1} \begin{bmatrix} C\theta_j \\ -C\alpha_{(j-1)j} S\theta_j \\ S\alpha_{(j-1)j} S\theta_j \end{bmatrix} \quad (2.11)$$

where S and C represent the trigonometric functions sine and cosine.

Now, let vector $\underline{R}_j$ define the position of the origin of the local frame in the fixed coordinate system. This vector is evaluated by a vector summation of the joint offsets and link lengths as shown here.

$$\underline{R}_j = S_{11}\underline{S}_1 + \sum_{\ell=2}^j (a_{(\ell-1)} \underline{a}_{(\ell-1)} + S_{\ell\ell}\underline{S}_\ell) \quad (2.12)$$

Double subscripts for joint offsets represent a fixed displacement, however, it must be replaced by S_ℓ where this value is not constant.

The position of a point, P , fixed in the j th reference frame is defined in terms of local coordinates by a vector $\underline{p}^{(j)} (x_p^{(j)}, y_p^{(j)}, z_p^{(j)})$. Vector $\underline{p}(x_p^*, y_p^*, z_p^*)$ designates the position vector of point P in terms of global coordinates, which can be determined by transforming $\underline{p}^{(j)}$ to the global frame. This transformation involves a translational transfer of the local frame's origin, and a rotational transfer of local coordinate axes.

$$\underline{p} = \underline{R}_j + T_j \underline{p}^{(j)} \quad (2.13)$$

2.2 Rate of Change of Orientation and Position

The rate of change of orientation of a rigid element is defined by a vector, $\underline{\omega} (\omega^x, \omega^y, \omega^z)$, representing the angular velocity of the element about the axes of a fixed coordinate system. The absolute angular velocity of link jk in a serial manipulator can be determined simply by summing the relative angular velocities between successive links preceding link jk ,

$$\underline{\omega}_{jk} = {}^{01}\underline{\omega}_{12} + {}^{12}\underline{\omega}_{23} + \dots + {}^{ij}\underline{\omega}_{jk}, \quad (2.14)$$

where ${}^{ij}\underline{\omega}_{jk}$ represents the angular velocity of link jk relative to link ij . The relative angular motion between links ij and jk is the angle θ_j , which is constrained to rotate about the $\underline{S}_j$ axis; therefore, the relative angular velocities may be written as

$${}^{ij}\underline{\omega}_{jk} = \dot{\theta}_j \underline{S}_j \quad (2.15)$$

where the dot represents the derivative with respect to time. Substituting Eqn. (2.15) into (2.14) results

$$\underline{\omega}_{jk} = \dot{\theta}_1 \underline{S}_1 + \dot{\theta}_2 \underline{S}_2 + \dots + \dot{\theta}_j \underline{S}_j \quad (2.16)$$

Now, Eqn. (2.15) can be written in a matrix form in terms of generalized motions, θ_n , and first order rotational geometric coefficients $[G_{jk}]$

$$\underline{\omega}_{jk} = [G_{jk}] \dot{\underline{\theta}} \quad (2.17)$$

where $[G_{jk}]$ is a $3 \times N$ (N denotes the number of degrees of freedom in the system) matrix and its column vectors are

$$[G_{jk}]_n = \frac{\partial \omega_{jk}}{\partial \dot{\theta}_n} \quad (2.18)$$

Notice that the subscript n designates the n th column of matrix $[G_{jk}]$ and notation $[G_{jk}]_n$ is chosen in place of the common notation $[G_{jk}]_{,n}$. From Eqns. (2.16) and (2.18) the column vectors of $[G_{jk}]$ are found to be

$$[G_{jk}]_n = \begin{cases} \underline{S}_n, & n \leq j; \text{ n-revolute} \\ 0, & \text{otherwise} \end{cases} \quad (2.19)$$

The rate of change of position or translational velocity of a point, P , fixed in link jk , is the time derivative of position vector $\underline{P}$

$$\begin{aligned} \underline{v}_P = \frac{d\underline{P}}{dt} = \frac{d\underline{R}_j}{dt} + \frac{d}{dt} (T_{jP}^{(j)}) &= \dot{\underline{s}}_1 \underline{s}_1 + \sum_{\ell=2}^j (a_{(\ell-1)\ell} \dot{a}_{(\ell-1)\ell} + \\ & s_{\ell} \dot{s}_{\ell} + \dot{s}_{\ell} \underline{s}_{\ell}) + \frac{d}{dt} (T_{jP}^{(j)}) \end{aligned} \quad (2.20)$$

The time rate of change of a vector $\underline{Q}$ fixed in a rigid body jk is defined [23] as

$$\frac{d\underline{Q}}{dt} = \underline{\omega}_{jk} \times \underline{Q} \quad (2.21)$$

Unit vectors, $\underline{a}_{(\ell-1)}$ and $\underline{s}_{\ell}$, are fixed in link $(\ell-1)$ and vector $T_{jP}^{(j)}$ is fixed in link jk . Considering this definition, we can rewrite Eqn. (2.20) in the form

$$\underline{v}_P = \sum_{n=1}^j \dot{\underline{s}}_n \underline{s}_n + \dot{\Theta}_n \underline{s}_n \times \left[\sum_{\ell=n+1}^j (a_{(\ell-1)\ell} \underline{a}_{(\ell-1)\ell} + s_{\ell\ell} \underline{s}_{\ell}) + T_{jP}^{(j)} \right] \quad (2.22)$$

Notice that the terms inside the second summation of Eqn. (2.22) represent a vector from the origin of n th local reference frame to point P , since

$$\underline{R}_n = S_{11}\underline{S}_1 + \sum_{\ell=2}^n (a_{(\ell-1)\ell} \underline{a}_{(\ell-1)\ell} + s_{\ell\ell}\underline{s}_{\ell}) \quad (2.23)$$

$$\underline{P} = S_{11}\underline{S}_1 + \sum_{\ell=2}^j (a_{(\ell-1)\ell} \underline{a}_{(\ell-1)\ell} + s_{\ell\ell}\underline{s}_{\ell}) + T_j \underline{P}^{(j)} \quad (2.24)$$

$$\underline{P} - \underline{R}_n = \sum_{\ell=n+1}^j (a_{(\ell-1)\ell} \underline{a}_{(\ell-1)\ell} + s_{\ell\ell}\underline{s}_{\ell}) + T_j \underline{P}^{(j)} \quad (2.25)$$

Eqn. (2.22) is then reduced to

$$\underline{V}_P = \sum_{n=1}^j [\dot{\underline{S}}_n \underline{S}_n + \dot{\Theta}_n \underline{S}_n \times (\underline{P} - \underline{R}_n)] \quad (2.26)$$

Introducing the first order translational geometric influence coefficients $[G_P]$ and generalized motions $\dot{\Phi}_n$, the velocity of point P can be written in the matrix form

$$\underline{V}_P = [G_P] \dot{\underline{\Phi}} \quad (2.27)$$

Recalling that $\dot{\Phi}_n$ is equal to $\dot{\Theta}_n$, if joint n is revolute, and it is equal to $\dot{S}_n$ if joint n is prismatic. By inspection of Eqn. (2.26), first order translational coefficients can be defined as

$$[G_p]_n = \frac{\partial V_p}{\partial \dot{\theta}_n} = \begin{cases} \underline{S}_n \times (\underline{P} - \underline{R}_n) , & n \leq j; \text{ n-revolute} \\ \underline{S}_n , & n \leq j; \text{ n-prismatic} \\ 0 & \text{otherwise.} \end{cases} \quad (2.28)$$

Rotational and translational velocities for parallel mechanisms can also be defined in terms of first order geometric coefficients; however, the definitions of these coefficients will be more complex than definitions introduced for serial mechanisms. The analysis of parallel mechanisms is beyond the scope of this paper, although the present work may be extended to include these systems.

2.3 Basis for the Mathematical Model

The motion of manipulators with rigid components is described by highly nonlinear, coupled, differential equations in terms of the actuator's motion. However, the vibratory motions of robot arms are low magnitude, oscillatory deformations about the mean motion equilibrium caused by small relative deflections of the elastic elements. Furthermore, the representative frequency of operation of the arm, ω , is kept significantly lower than the system's fundamental natural frequency. This prevents the dynamic coupling between the small and gross motion of the arm since many vibration cycles will occur about the

mean motion before any significant change in the arm configuration occurs (the detailed discussion of coupling between small and gross motion is presented in Chapter IV). In addition, the small motion assumption causes the nonlinear terms in the equations of motion to become insignificant. It is then sufficient to describe the vibratory motion of the manipulator by a set of linear, coupled, differential equations in terms of the perturbed motion of the arm about its stationary configurations.

A lumped mass arm model is used for tractability. Not only does this produce a set of ordinary differential equations of motion instead of partial differential equations, it also provides an accurate description of system energy content. The distributed mass model is not chosen since the manipulator's links, in general, have complex geometric shapes. Because of this complexity in geometry, the distributed mass theory for beams fails to describe the mass content of the link accurately. Furthermore, the kinetic energy contribution of elements, such as electrical motors, hydraulic actuators, and drive trains, which are very significant, must be included in the total energy content of the system. This requires a mixed distributed lumped formulation, leading to a set of partial and ordinary differential equations describing the system's motion. Dynamic analysis of flexible manipulators is a complex problem; such formulation creates even more complication in this analysis.

The finite element method can be utilized to describe mass distribution of a complex shaped element by representing the element as a series of small uniform elements connected to each other. A system's degree of freedom will increase rapidly by using this method, which will lead to a large system of equations describing the manipulator's motion.

In conclusion, the lumped mass model provides an accurate representation for most precision manipulators since the arms do not generally have long, flexible links, such as those found in the shuttle manipulator. An experimental procedure will be introduced in Chapter V to identify the concentrated mass and stiffness parameters, providing an accurate description of the manipulator's mass and flexibility content.

2.4 Dynamic Model of Serial Manipulators with Flexible Joints

For many manipulator designs, most of the deformation energy appears in the actuator and drive train components. Therefore, an initial model considers flexibility at the joints in the direction of relative joint motion only. The corresponding mass of link j_k is represented by an inertia ellipsoid [23] centered at point C_j with principal axes parallel to the j th local reference frame

as shown in Fig. 2.2. Vector $\underline{C}_j$ defines the undeformed position of mass center in terms of the global reference frame.

Consider a serial manipulator with rigid components. The time derivative of vector $\underline{C}_j$ is obtained from Eqn.

(2.27)

$$\dot{\underline{C}}_j = [G_{cj}] \dot{\underline{\Phi}} . \quad (2.29)$$

Using the mathematical definition of the time derivative, Eqn. (2.29) can be rewritten as

$$\frac{\Delta \underline{C}_j}{\Delta t} = [G_{cj}] \frac{\Delta \underline{\Phi}}{\Delta t} \quad (2.30)$$

Canceling Δt from both sides leads to

$$\Delta \underline{C}_j = [G_{cj}] \Delta \underline{\Phi} . \quad (2.31)$$

This equation describes the variation in the position of mass center $\underline{C}_j$, due to the small relative motion of the joints. Vector $\Delta \underline{\Phi}$ portrays the elastic deformation of the

joints about their axes, and may be replaced by a set of relative perturbed motions, β_n , as shown in Fig. 2.2. The deformed position of point C_j is defined by a vector $\hat{\underline{C}}_j$, (the hat throughout this dissertation designates a vector which is displaced from its original line of action due to the deformation of the flexible components). Considering Fig. 2.2, it is evident that

$$\hat{\underline{C}}_j = \underline{C}_j + \Delta \underline{C}_j = \underline{C}_j + [G_{Cj}] \underline{\beta} . \quad (2.32)$$

The equations of motion for a mechanical system with N degree of freedom can be obtained from Lagrangian equations

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{q}_n} \right) - \frac{\partial KE}{\partial q_n} + \frac{\partial PE}{\partial q_n} = Q_n \quad (2.33)$$

where KE and PE are the total kinetic and potential energies of the mechanism. The generalized position coordinates are designated by q_n and their corresponding generalized forces are Q_n .

The velocity of mass center C_j is found by differentiating Eqn. (2.32) with respect to time

$$\dot{\underline{\hat{C}}}_j = \frac{d\underline{\hat{C}}_j}{dt} = \frac{d\underline{C}_j}{dt} + \left(\frac{d}{dt} [G_{cj}] \right) \underline{\beta} + [G_{cj}] \underline{\dot{\beta}}. \quad (2.34)$$

Recalling that the manipulator arm is in a fixed configuration, and also assuming the variation of the first order coefficients due to the deflection of the arm is negligible, we can rewrite Eqn. (2.34) in a simplified form as

$$\dot{\underline{\hat{C}}}_j = [G_{cj}] \underline{\dot{\beta}}. \quad (2.35)$$

The absolute angular velocity of the link jk can be obtained from Eqn. (2.17),

$$\underline{\hat{\omega}}_{jk} = [G_{jk}] \underline{\dot{\beta}}. \quad (2.36)$$

The manipulator's kinetic energy is obtained from the sum of the kinetic energies produced by the motion of inertia mass ellipsoids

$$KE = \sum_{j=1}^N \frac{1}{2} \{ M_j \underline{\hat{C}}_j^T \underline{\hat{C}}_j + \underline{\hat{\omega}}_{jk}^T [\Pi_{jk}] \underline{\hat{\omega}}_{jk} \} \quad (2.37)$$

where M_j is the total mass of link jk concentrated at point C_j , and $[\Pi_{jk}]$ designates the inertia tensor about the centroid for link jk in terms of the global coordinates. The values in this inertia tensor depend on the orientation of link jk . However, the inertia tensor about the centroid, defined in terms of the local coordinate axes, $[\Pi_{jk}^{(j)}]$, is a constant diagonal matrix for each link, which depends only on mass distribution of the link. To derive an expression for $[\Pi_{jk}]$, recognize that the kinetic energy contribution is independent of the coordinate system so

$$\hat{\underline{\omega}}_{jk}^T [\Pi_{jk}] \hat{\underline{\omega}}_{jk} = (\hat{\underline{\omega}}_{jk}^{(j)})^T [\Pi_{jk}^{(j)}] \hat{\underline{\omega}}_{jk}^{(j)} \quad (2.38)$$

The absolute angular velocity of link jk in terms of the global coordinates can be found from

$$\hat{\underline{\omega}}_{jk} = T_j \hat{\underline{\omega}}_{jk}^{(j)}. \quad (2.39)$$

Since T_j is an orthogonal matrix, the inverse of this matrix is equal to its transpose. So we can write

$$\hat{\underline{\omega}}_{jk}^{(j)} = T_j^T \hat{\underline{\omega}}_{jk}. \quad (2.40)$$

Substituting Eqn. (2.40) into (2.38) results in

$$\hat{\underline{\omega}}_{jk}^T [\Pi_{jk}] \hat{\underline{\omega}}_{jk} = \hat{\underline{\omega}}_{jk}^T T_j [\Pi_{jk}^{(j)}] T_j^T \hat{\underline{\omega}}_{jk}. \quad (2.41)$$

From which, the inertia tensor, $[\Pi_{jk}]$, is defined as

$$[\Pi_{jk}] = T_j [\Pi_{jk}^{(j)}] T_j^T. \quad (2.42)$$

To express the system's kinetic energy in terms of the first order influence coefficients, Eqn. (2.37) is rewritten using Eqns. (2.35) and (2.36)

$$\begin{aligned} KE &= \frac{1}{2} \sum_{j=1}^N M_j \dot{\underline{\beta}}^T [G_{cj}]^T [G_{cj}] \dot{\underline{\beta}} + \dot{\underline{\beta}}^T [G_{jk}]^T [\Pi_{jk}] [G_{jk}] \dot{\underline{\beta}} \\ &= \frac{1}{2} \dot{\underline{\beta}}^T [I^*] \dot{\underline{\beta}} \end{aligned} \quad (2.43)$$

The equivalent inertia matrix of the system is represented by the $N \times N$ matrix $[I^*]$, which defines the effective mass of the system as seen at each joint. The components of this matrix are obtained from the general equation

$$[I^*]_{m;n} = \sum_{j=1}^N M_j [G_{cj}]_m^T [G_{cj}]_n + [G_{jk}]_m^T [\Pi_{jk}] [G_{jk}]_n \quad (2.44)$$

where $[I^*]_{m;n}$ designates the scalar component of the inertia matrix located at m th row and n th column.

The potential energy of a manipulator in a deflected configuration is the sum of the strain energies of the springs describing the arm's flexible components

$$PE = \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^N K_{ij} \beta_i \beta_j = \frac{1}{2} \underline{\beta}^T [K^*] \underline{\beta} \quad (2.45)$$

where K_{ij} are the stiffness coefficients and $[K^*]$ is the equivalent stiffness matrix. Recalling that the manipulator's deflection is due to the relative deformation of joints in the direction of joint motion. The system's flexible components can be represented by a set of torsional springs attached to the revolute joint axes, and translational springs attached to the prismatic joint axes (Fig. 2.2). The energy content of each spring is due to the displacement of the corresponding joint only; therefore, there is no coupling in the system's potential energy, or

$$K_{ij} = \begin{cases} K_{jj} & i=j \\ 0 & i \neq j. \end{cases} \quad (2.46)$$

Notice that the gravitation energy will not be included since the effect of this energy can be analyzed statically.

Selecting β_n as the generalized coordinates, the Lagrange equations for free vibration of an undamped mechanical system can be rewritten in the form

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{\beta}_n} \right) - \frac{\partial KE}{\partial \beta_n} + \frac{\partial PE}{\partial \beta_n} = 0 \quad n = 1, 2, \dots, N. \quad (2.47)$$

Substituting the final expressions for the kinetic and potential energies into Eqn. (2.47), the equation of motion can be written as

$$\frac{d}{dt} ([I^*]_n; \dot{\underline{\beta}}) - \frac{1}{2} \dot{\underline{\beta}}^T \left(\frac{\partial}{\partial \beta_n} [I^*] \right) \dot{\underline{\beta}} + [K^*]_n; \underline{\beta} = 0 \quad (2.48)$$

The subscript, n , designates the n th row of a matrix. The first term on the right side of Eqn. (2.48) can be expanded as shown here

$$\frac{d}{dt} ([I^*]_n; \dot{\underline{\beta}}) = \left(\frac{d}{dt} [I^*]_n; \right) \dot{\underline{\beta}} + [I^*]_n; \ddot{\underline{\beta}}. \quad (2.49)$$

Recalling that $[I^*]$ is a function of the local mass parameters and first order geometric coefficients. Also, these coefficients are assumed to be invariant with respect to small displacements β_n , the second term on the left side of Eqn. (2.48) and the first term on the right side of Eqn. (2.49) can be neglected. The n th equation of motion is then reduced to

$$[I^*]_n, \ddot{\underline{\beta}} + [K^*]_n, \underline{\beta} = 0. \quad (2.50)$$

Next, the complete system's equations of motion can be written in the matrix form

$$[I^*] \ddot{\underline{\beta}} + [K^*] \underline{\beta} = 0. \quad (2.51)$$

Vector $\underline{\beta}$ describes the oscillatory motion of the manipulator's joint angles, which can be defined by a harmonic equation as

$$\underline{\beta} = \underline{B} e^{i\omega_r t} \quad (2.52)$$

where vector $\underline{B}$ is the magnitude and ω_r is the frequency of the joints' vibration. Substituting Eqn. (2.52) into the equations of motion gives

$$\{-\omega_r^2 [I^*] + [K^*]\} \underline{B} e^{i\omega_r t} = 0. \quad (2.53)$$

For nontrivial $\underline{B}$, the coefficient matrix must be singular so its determinant must be zero.

$$-\omega_r^2 [I^*] + [K^*] = 0 \quad (2.54)$$

Eqn. (2.54) can be rewritten in the form

$$[K^*]^{-1} [I^*] - \frac{1}{\omega_r^2} [I] = 0 \quad (2.55)$$

where $[I]$ is an $N \times N$ identity matrix. This is a standard eigenvalue problem, which can be solved to determine the system's natural frequencies, ω_r , and the corresponding eigenvectors $\psi^{(r)}$. Notice that $[K^*]$ is a diagonal matrix and simple to invert. In general, there is no closed form solution to Eqn. (2.55) and it must be solved numerically. There are several numerical techniques which can be utilized to evaluate the eigenvalues and eigenvectors of a system of equations. A matrix iteration scheme, using the power method in conjunction with the sweeping technique [24-27], is found to be most practical to determine the first few eigenvalues and eigenvectors of a large system of equations. Also, an iteration scheme based on the QR method [27-28] is best suited to evaluate all of the system's natural frequencies and the corresponding mode shapes.

2.5 The Secondary Flexible Components of A Serial Manipulator

The dynamic model of a manipulator with flexible joints may not describe the system's deformations adequately if the deformation energies contributed by other flexible components are significant. The most prominent deflection modes of a joint-link combination (Fig. 2.3a) are a) the motion about the joint axis, as discussed previously; b) the deformation of the joint bearings about two orthogonal axes perpendicular to the joint axis; and c) the torsional twisting of the link about its major axis. The joint deflection is described by an input, β_1 , about joint axis, $\underline{S}_1$. Two pseudo-inputs, β_2 and β_3 , are introduced to describe the bearing deflections and the torsional deformation of the link (pseudo-inputs are imaginary inputs that produce small motions only). Parameter, β_2 , represents the combined link deflection and joint bearing deflection perpendicular to $\underline{S}_1$ about $\underline{S}_2$ (Fig. 2.3b). The combined bearings' deflection and link's torsional deformation about an axis, $\underline{S}_3$, perpendicular to $\underline{S}_1$ and $\underline{S}_2$, are described by the pseudo-input β_3 . The mass content of this joint-link combination is lumped at the imaginary link, a_{34} , so that the motion of β_1 , β_2 , and β_3 will contribute to the kinetic energy of this

component (Fig. 2.3b). Now, the system's equations of motion can be obtained by following the same procedure as described in Section 2.4. The length of vector $\underline{\beta}$ will increase to at most $3N$ (often, fewer than three inputs are needed for a sufficiently accurate representation). The components of inertia matrix, $[I^*]$, can be obtained from Eqn. (2.44), but the size of this matrix will increase by a factor of three. Notice that the $3N \times 3N$ stiffness matrix $[K^*]$ will remain diagonal.

2.6 Dynamic Model of Serial Manipulators with Flexible Links

This section expands the formulation of Section 2.4 to include the flexibilities of the manipulator's links in addition to those of joint components. A general joint-link combination is shown in Fig. 2.4a. This component is shown in its rigid-body position (solid lines), as well as in its elastically deformed configuration (dashed lines). The elastic deformations of the joint-link combination may be completely described by the input, β_1 (joint deflection), and six pseudo-inputs, β_2 through β_7 (link deformations), as shown in Fig. 2.4b. The flexibility of each joint-link element can be represented by a 7×7 stiffness matrix, $[K_{jk}]$, where subscript, jk , designates link jk . The length of vector

$\underline{\beta}$, will increase by a factor of seven. Considering the strains associated with displacements, $\underline{\beta}$, the system's potential energy is

$$PE = \frac{1}{2} \underline{\beta}^T [K^*] \underline{\beta} \quad (2.56)$$

where $[K^*]$ is a $7N \times 7N$ matrix of the form

$$[K^*] = \begin{bmatrix} [K_{23}] & & & \\ & [K_{23}] & & 0 \\ & & [K_{34}] & \\ 0 & & & [K_{N(N+1)}] \end{bmatrix} \quad (2.57)$$

Recalling that the size of the inertia matrix will increase by a factor of seven, its scalar component can be obtained from Eqn. (2.44). Finally, the manipulator's equations of motion are derived using the formulation of Section 2.4.

For a general six degree of freedom flexible manipulator arm, the motion is described by forty-two differential equations. Thus, it may be difficult to compute all the system's natural frequencies. However, higher natural frequencies have little or no effect on system vibration, since they are higher and smaller in magnitude than the fundamental frequency. It is then

adequate to evaluate the first few eigenvalues using the power method.

2.7 Improvement of System Design

The performance and operating speed of a system can be improved by increasing its fundamental natural frequency. This will reduce the robot's flexibility, leading to less deflection of the arm due to the applied loads and inertia forces.

An approximation for the lowest eigenvalue of a system of equations is given by Rayleigh's quotient

$$\omega_1^2 \cong \frac{(\underline{u})^T [K^*] (\underline{u})}{(\underline{u})^T [I^*] (\underline{u})} \quad (2.58)$$

An estimate for the fundamental frequency can be obtained from Eqn. (2.58) if the arbitrary vector $\underline{u}$ resembles the first mode of vibration. If the vector $\underline{u}$ coincides with the system fundamental mode $\underline{\psi}^{(1)}$, we can write

$$\omega_1^2 = \frac{(\underline{\psi}^{(1)})^T [K^*] (\underline{\psi}^{(1)})}{(\underline{\psi}^{(1)})^T [I^*] (\underline{\psi}^{(1)})} = \frac{\bar{K}_1}{\bar{M}_1} \quad (2.59)$$

Parameters, $\bar{K}_1$ and $\bar{M}_1$, designate the modal stiffness and modal mass of the system. Clearly, the magnitude of ω_1 , can be increased by decreasing $\bar{M}_1$ and/or increasing $\bar{K}_1$. To demonstrate the effect of the local stiffness and mass parameters on the system fundamental frequency, $\bar{K}_1$ and $\bar{M}_1$ must be defined explicitly in terms of these parameters, as follows:

$$\bar{K}_1 = \sum_{i=1}^N \sum_{j=1}^N K_{ij} \psi_i^{(1)} \psi_j^{(1)} \quad (2.60)$$

$$\bar{M}_1 = \sum_{j=1}^N M_j D_j \quad (2.61)$$

where $\psi_i^{(1)}$ denotes the i th component of $\underline{\psi}^{(1)}$ and

$$D_j = \sum_{j=1}^N \sum_{\ell=1}^N [F_j]_{i;\ell} \quad (2.62)$$

$[F_j]$ is an $N \times N$ matrix, which is constructed from

$$[F_j] = [G\psi_{cj}]^T [G\psi_{cj}] + [G\psi_{jk}]^T [E_{jk}] [G\psi_{jk}]. \quad (2.63)$$

The column vectors of $[G\psi]$ matrices are defined as

$$[G\psi_{cj}]_n = \psi_n^{(1)} [G_{cj}]_n \quad (2.64)$$

$$[G\psi_{jk}]_n = \psi_n^{(1)} [G_{jk}]_n. \quad (2.65)$$

The 3×3 matrix, $[E_{jk}]$, is obtained from

$$[E_{jk}] = [T_j] [e_{jk}] [T_j]^T \quad (2.66)$$

where e_{jk} is a 3×3 diagonal matrix with constant components. This matrix is defined in terms of the local mass, M_j . Assuming that the local inertia tensor of each link is proportional to its mass, we can write

$$[e_{jk}] = \frac{1}{M_j} [\pi_{jk}^{(j)}]. \quad (2.67)$$

In general, the scalar components of $[e_{jk}]$ fluctuate as the value of M_j varies. However, to express $\bar{M}_1$ in terms of the local masses, M_j , only, we assume that $[e_{jk}]$ remains constant. Now, by evaluating the coefficients of Eqns. (2.60) and (2.61), it is very simple to investigate the effect of each local mass and stiffness on the system fundamental frequency.

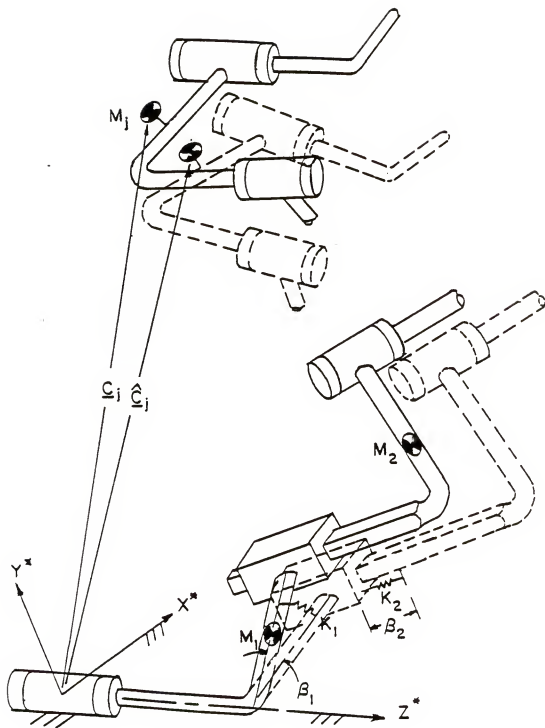
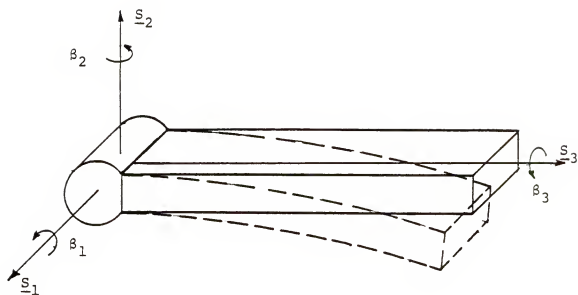
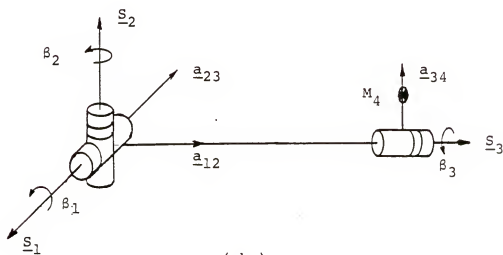


Figure 2.2. Manipulator with Elastic Joints Displaced from their Rigid-Body Position.



(a)



(b)

Figure 2.3. Secondary Deformations of Joint-Link Combination.

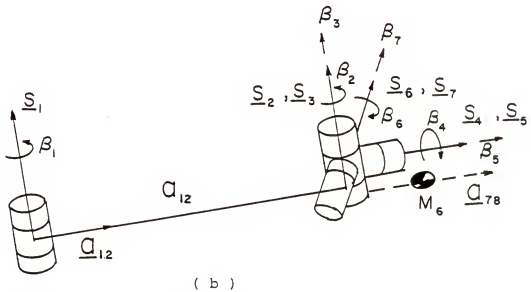
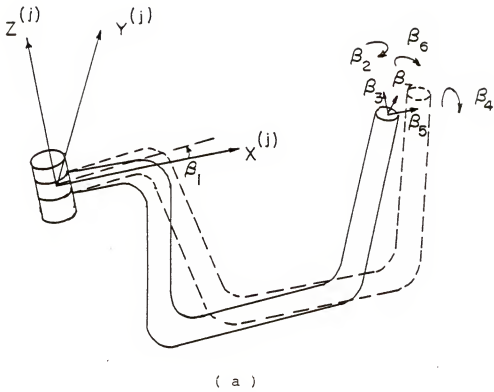


Figure 2.4. General Deformation of Joint-Link Combination.

CHAPTER III

ANALYSIS OF MANIPULATOR DYNAMIC RESPONSE

The response of a manipulator arm is the behavior of the system (the motion of a point in the system at any time) when subjected to a certain excitation. This excitation is caused by transient phenomena such as dynamic shocks associated with the arm motion. In addition, there is a certain harmonic content related to the loading condition of the manipulator's terminal device which may excite one or more vibrational modes of the arm.

Using the mathematical model presented in Chapter II, it is possible to formulate the forced vibration of a manipulator arm under a general exciting condition. The response of the arm can be represented by N coupled, linear, ordinary differential equations in terms of the generalized coordinates β_n . These coordinates can be obtained from a solution of the coupled equations, but this procedure becomes progressively cumbersome as the degree of freedom of the system increases. A more suitable approach is to decouple the system of equations using the concept of modal analysis.

The transfer function is a general concept relating the system response to the excitation in the frequency domain which contains all the information concerning the system vibration characteristics. To represent the manipulator's characteristics, its transfer functions are obtained by exciting the mechanism with a harmonic force of variable frequency.

The equations of motion describing the system oscillation have been linearized by neglecting the variation of the first order geometric coefficients due to small displacements β_n . It is appropriate to investigate the significance of the nonlinear terms by extending the previous formulation to include the Coriolis and centripetal accelerations in the equations of motion.

3.1 Forced Vibration

In the most general case, several time varying forces and torques can be applied to each link of the manipulator arm. These applied loads can be represented by N generalized forces acting over the generalized coordinates β_n . Representing the generalized forces as T_n , the equations of motion can be obtained from Lagrange's equations as follows

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{\beta}_n} \right) - \frac{\partial KE}{\partial \beta_n} + \frac{\partial W^D}{\partial \dot{\beta}_n} + \frac{\partial PE}{\partial \beta_n} = T_n \quad (3.1)$$

where W^D is the work done by a special type of friction force arising from motion in a viscous medium. The generalized forces can be defined in terms of the first order coefficients and applied loads from the principle of virtual work, as described here.

Consider the resultant of the applied loads on link jk of the manipulator as a force, $\underline{f}_p$, applied to point P in the link and a moment, $\underline{m}_{jk}$, applied to the link. The virtual work due to the applied loads on link jk can be defined as

$$\delta W = \underline{f}_p^T \underline{v}_p \delta t + \underline{m}_{jk}^T \underline{\omega}_{jk} \delta t \quad (3.2)$$

The virtual displacements of point P and link jk are represented by $\underline{v}_p \delta t$ and $\underline{\omega}_{jk} \delta t$, where δt is a virtual time increment. However, the virtual work can also be regarded as the product of N generalized forces $T_n^{(jk)}$ acting over the virtual displacements $\dot{\phi}_n \delta t$.

$$\delta W = \sum_{n=1}^N T_n^{(jk)} \dot{\phi}_n \delta t = (\underline{T}^{(jk)})^T \underline{\dot{\phi}} \delta t \quad (3.3)$$

Notice that $T_n^{(jk)}$ designates the equivalent force acting over the n th input due to the applied loads on link

jk. Now, Eqn. (3.2) can be rewritten in terms of the geometric coefficients using Eqns. (2.35) and (2.36)

$$\delta W = \underline{f}_p^T [\underline{G}_p] \dot{\underline{\phi}} \delta t + \underline{m}_{jk}^T [\underline{G}_{jk}] \dot{\underline{\phi}} \delta t \quad (3.4)$$

so that, comparing Eqns. (3.3) and (3.4), we can conclude that the generalized forces have the form

$$(\underline{T}^{(jk)})^T = \underline{f}_p^T [\underline{G}_p] + \underline{m}_{jk}^T [\underline{G}_{jk}] . \quad (3.5)$$

The total equivalent forces $\underline{T}$ are found by summing $\underline{T}^{(jk)}$ for each link with applied loads, or

$$\underline{T} = \sum_{j=1}^N \underline{T}^{(jk)} . \quad (3.6)$$

The friction forces are nonconservative forces and are assumed to be proportional to the velocities. Therefore, the work done by these forces can be obtained from Rayleigh's dissipation function

$$W^D = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N C_{ij} \dot{\beta}_i \dot{\beta}_j \quad (3.7)$$

where C_{ij} represents the damping coefficients. This equation can be rewritten in a matrix form as

$$W^D = \frac{1}{2} \dot{\underline{\beta}}^T [C^*] \dot{\underline{\beta}} . \quad (3.8)$$

The equivalent damping matrix is defined by $[C^*]$. Introducing Eqns. (2.43), (2.45) and (3.8) in (3.1), we obtain a set of N coupled, ordinary, differential equations describing the response of the manipulator arm. The equations of motion can be written in the matrix form

$$[I^*] \ddot{\underline{\beta}} + [C^*] \dot{\underline{\beta}} + [K^*] \underline{\beta} = \underline{T} . \quad (3.9)$$

The system of equations (3.9) can be solved for the generalized coordinates β_n which represents the oscillation of the arm at any time t . But, this is possible only if the excitation is given in a deterministic form as a time dependent function.

3.2 Decoupled Equations of Motion

It is not practical to solve the coupled equations of motion simultaneously when the system has many degrees of freedom. In this case, a different approach can be used to give the equations of motion a simpler form by expressing the equations in a new set of coordinates. This is accomplished by decoupling the equations of motion using a linear coordinate transformation. The linear

transformation is obtained by assuming that the response is a superposition of the normal modes of the system multiplied by the corresponding time dependent modal coordinate. To use this procedure which is known as modal analysis, it is necessary first to obtain the system eigenvalues and eigenvectors from the eigenvalue problem described by Eqn. (2.55). Using the superposition theorem, the response can be described by the linear transformation

$$\underline{\beta} = [\Psi]\underline{\eta} \quad (3.10)$$

where $\underline{\eta}$ is a column matrix consisting of a set of coordinates referred to as modal coordinates. The modal matrix $[\Psi]$ is an $N \times N$ matrix and its column vectors are defined by the equation

$$[\Psi]; r = \underline{\psi}^{(r)} . \quad (3.11)$$

The components of $[\Psi]$ are independent of time, so the derivatives of the generalized coordinates with respect to time can be represented by the equations

$$\dot{\underline{\beta}} = [\Psi] \dot{\underline{\eta}} \quad (3.12)$$

$$\ddot{\underline{\beta}} = [\Psi] \ddot{\underline{\eta}} . \quad (3.13)$$

Introducing Eqns. (3.10), (3.12) and (3.13) into (3.9), we obtain

$$[I^*] [\psi] \ddot{\underline{n}} + [C^*] [\psi] \dot{\underline{n}} + [K^*] [\psi] \underline{n} = \underline{T} . \quad (3.14)$$

Premultiplying both sides of this equation by $[\psi]^T$, we can write

$$[\psi]^T [I^*] [\psi] \ddot{\underline{n}} + [\psi]^T [C^*] [\psi] \dot{\underline{n}} + [\psi]^T [K^*] [\psi] \underline{n} = [\psi]^T \underline{T} . \quad (3.15)$$

But, from the orthogonality condition of the normal modes we have

$$(\underline{\psi}^{(r)})^T [I^*] (\underline{\psi}^{(s)})^T = \begin{cases} \bar{M}_r & , \quad r=s \\ 0 & , \quad r \neq s \end{cases} \quad (3.16)$$

$$(\underline{\psi}^{(r)})^T [K^*] (\underline{\psi}^{(s)})^T = \begin{cases} \bar{K}_r & , \quad r=s \\ 0 & , \quad r \neq s \end{cases} \quad (3.17)$$

where $\bar{M}_r$ and $\bar{K}_r$ are the modal mass and modal stiffness of the system associated with the r th mode of vibration. Therefore, the system inertia and stiffness matrices corresponding to the modal coordinates are diagonal and can be defined by the equations

$$[\bar{M}] = [\psi]^T [I^*] [\psi] \quad (3.18)$$

$$[\bar{K}] = [\psi]^T [K^*] [\psi] . \quad (3.19)$$

In general, the damping matrix associated with the modal coordinates is not diagonal. However, the damping factors are generally small and matrix $[C^*]$ can be assumed to be proportional to the system inertia and stiffness matrices, or

$$[C^*] \approx v_1 [K^*] + v_2 [I^*] . \quad (3.20)$$

The error introduced by this approximation for the robotic manipulator is expected to be insignificant. Hence, the damping matrix in modal coordinates can be defined by a diagonal matrix as

$$[\bar{C}] = [\Psi]^T [C^*] [\Psi] . \quad (3.21)$$

Now, using the definitions for modal parameters we can obtain N independent equations in terms of the modal coordinates

$$\bar{M}_r \ddot{\eta}_r + \bar{C}_r \dot{\eta}_r + \bar{K}_r \eta_r = \bar{N}_r \quad (3.22)$$

where $\bar{N}_r$ represent the generalized forces associated with the modal coordinates, which are given by the equation

$$\bar{N}_r = (\underline{\psi}^{(r)})^T \underline{T} . \quad (3.23)$$

Dividing both sides of Eqn. (3.22) by $\bar{M}_r$, we obtain

$$\ddot{\eta}_r + 2 \zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = \frac{\bar{N}_r}{\bar{M}_r} \quad (3.24)$$

where by definition

$$\zeta_r = \frac{\bar{C}_r}{(2 \bar{K}_r \bar{M}_r)^{1/2}} \quad (3.25)$$

$$\omega_r = (\bar{K}_r / \bar{M}_r)^{1/2} . \quad (3.26)$$

The solution of Eqn. (3.24) can be readily obtained by means of the Laplace transformation. This solution can be expressed as

$$\eta_r = \frac{1}{\omega_{dr} \bar{M}_r} \int_0^t N_r(\tau) e^{-\zeta_r \omega_r (t-\tau)} \sin \omega_{dr} (t-\tau) d\tau + \frac{\eta_r(0)}{(1-\zeta_r^2)^{1/2}} \cos(\omega_{dr} t - \gamma_r) + \frac{\dot{\eta}_r(0)}{\omega_{dr}} \sin \omega_{dr} t . \quad (3.27)$$

Parameters, $\eta_r(0)$ and $\dot{\eta}_r(0)$, represent the initial displacement and velocity corresponding to the r th modal coordinate; γ_r , and ω_{dr} are defined by the equations

$$\gamma_r = \tan^{-1} \frac{\zeta_r}{(1-\zeta_r^2)^{1/2}} \quad (3.28)$$

$$\omega_{dr} = (1 - \zeta_r^2)^{1/2} \omega_r . \quad (3.29)$$

Clearly, the modal analysis is a convenient procedure for computing the manipulator's response. However, determination of all system eigenvectors becomes complicated as the degree of freedom of the system increases. This difficulty can be overcome by approximating the system response by a superposition of the first few normal modes, which requires the calculation of the first few eigenvectors only. The error introduced by this approximation is negligible, since the higher natural frequencies are generally much higher than the fundamental frequency, and the contribution of their associated modes of vibration to the system response is very small.

3.3 Dynamic Response in the Frequency Domain

The characteristics of a vibratory system are fully described by the concept of a transfer function. This function represents the magnitude of oscillation at any frequency, when the system is subjected to a periodic force. Therefore the behavior of the system can be predicted if the frequency content of the time dependent exciting function is known. For instance, consider an impulsive force and its Fourier transformation as shown in Fig. 3.1. The Fourier transform of this function suggests that an impulsive force contains an infinite range of frequencies. Consequently, this force will excite every

vibration mode of a system when applied to the system. But, the forcing function of Fig. 3.2 will excite only those vibration modes which are within the frequency content of this force.

To obtain the manipulator's transfer functions, a harmonic force $\underline{F}_H e^{i\omega t}$ is applied to a point H located on the manipulator's end-effector. The corresponding generalized forces can then be acquired from Eqn. (3.5)

$$\underline{T}^T = \underline{F}_H^T [G_H] e^{i\omega t} . \quad (3.30)$$

Substituting Eqn. (3.30) into (3.23), we obtain

$$\bar{N}_r = |\bar{N}_r| e^{i\omega t} \quad (3.31)$$

where

$$|\bar{N}_r| = (\underline{\psi}^{(r)})^T [G_H]^T \underline{F}_H . \quad (3.32)$$

Introducing Eqn. (3.31) in (3.24), we can write

$$\ddot{\eta}_r + 2 \zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = |\bar{N}_r| e^{i\omega t} . \quad (3.33)$$

A solution of this differential equation is

$$\eta_r = |\eta_r| e^{i\omega t} . \quad (3.34)$$

Substituting Eqn. (3.34) into (3.33) and rearranging the resultant equation, we obtain

$$\frac{|\underline{\eta}_r|}{|\underline{N}_r|} = \bar{h}_{rr}(\omega) \quad (3.35)$$

where $\bar{h}_{rr}(\omega)$ represents the direct transfer function of the arm associated with the r th modal coordinate and is defined by the equation

$$\bar{h}_{rr}(\omega) = \frac{1}{\bar{K}_r [1 - (\omega/\omega_r)^2 + 2i\zeta_r (\omega/\omega_r)]} \quad (3.36)$$

The magnitude of the generalized coordinates $\underline{B}$ can be obtained from Eqn. (3.10)

$$\underline{B} = [\Psi] |\underline{\eta}| \quad (3.37)$$

We can also find the direct transfer function, $h_{nn}(\omega)$, corresponding to the n th generalized coordinate of the arm

$$\bar{h}_{nn}(\omega) = \frac{B_n}{|T_n|} \quad (3.38)$$

3.4 System Response Due to Dynamic Shocks

Any discontinuity in a time derivative of the motion of a mechanical system will create a shock in the dynamic operation of the system [2]. Such shocks may excite one or more modes of vibration of the system causing unwanted noise, fatigue, and reduced precision of operation. These shocks are most undesirable when they occur in the first and second time derivative of the system's motion (velocity and acceleration) and must be avoided if possible. A discussion of the manipulator response due to discontinuity in the velocity and acceleration curves is presented in this section.

Velocity shocks are commonly generated when starting or stopping the arm motion. Suppose that the velocity of the end-effector produced by the input commands is constant throughout the motion interval (Fig. 3.3a). The acceleration associated with this velocity (Fig. 3.3b) will be very high at the boundaries of the motion interval due to the velocity shocks at these points. Consequently, impulsive forces are generated at the boundaries by the inertia of the arm associated with this acceleration. Now, consider an impulsive force, $\underline{f}_H \delta(t)$, applied to a point H on the manipulator's terminal device. Using Eqns. (3.5) and (3.23), the effective force corresponding to the r th modal coordinate of the arm can be obtained

$$\bar{N}_r = |\bar{N}_r| \delta(t) \quad (3.39)$$

where

$$|\bar{N}_r| = (\psi^{(r)})^T [G_H]^T \underline{f}_H. \quad (3.40)$$

Substituting Eqn. (3.40) into (3.27) and neglecting the initial conditions, we can write

$$\eta_r = \frac{|\bar{N}_r|}{\omega_{dr} M_r} e^{-\zeta_r \omega_{dr} t} \sin \omega_{dr} t \quad (3.41)$$

This equation describes the behavior of the arm in the modal coordinates, from which the system response in the generalized coordinates can be obtained

$$\underline{\beta} = [\Psi] \underline{\eta}. \quad (3.42)$$

Furthermore, the behavior of any point P on the system is described by the equation

$$\Delta \underline{p} = [G_p] \underline{\beta}. \quad (3.43)$$

Discontinuities in the acceleration curve generate similar discontinuities in the corresponding inertia force. The exciting force produced by an acceleration

shock can be represented by a step function $\underline{f}_H \mu(t)$ applied to point H, where

$$\mu(t) = \begin{cases} 0 & , \quad t < 0 \\ 1 & , \quad t \geq 0 \end{cases} \quad (3.44)$$

The modal coordinates are then obtained from Eqn. (3.27)

$$\eta_r = \frac{|\bar{N}_r|}{\bar{K}_r} [1 - e^{-\zeta_r \omega_{dr} t} (\cos \omega_{dr} t + \frac{\zeta_r}{(1 - \zeta_r^2)^{1/2}} \sin \omega_{dr} t)]. \quad (3.45)$$

Notice that $|\bar{N}_r|$ is defined by Eqn. (3.40). The generalized coordinates β_n and the motion of any point P on the arm can be found from equations (3.42) and (3.43).

3.5 Nonlinearities in the Equations of Motion

The response of a manipulator arm was described in Section 3.1 by a set of linear differential equations. This was accomplished by neglecting the variation of the first order geometric coefficients due to small displacements β_n . Further refinement of the vibration formulation is needed to investigate the influence of nonlinear terms on the response of the arm.

Consider the instantaneous configuration of the manipulator arm shown in Fig. 3.4 (dashed lines) as it

vibrates about its rigid body position (solid lines). The total relative motion of each joint is defined by a parameter $\hat{\phi}_n(t)$, where

$$\hat{\phi}_n(t) = \phi_n + \beta_n(t) . \quad (3.46)$$

Each of the joint axes and link axes is displaced from its original line of action due to the deformation of the flexible components. These axes are defined by vectors $\hat{\underline{S}}_j$ and $\hat{\underline{a}}_{jk}$, respectively. Furthermore, the local coordinates $(\hat{x}^{(j)}, \hat{y}^{(j)}, \hat{z}^{(j)})$ corresponding to each link are chosen such that $\hat{x}^{(j)}$ and $\hat{z}^{(j)}$ lie along $\hat{\underline{a}}_{jk}$ and $\hat{\underline{S}}_j$. Thus, the rotation matrix $\hat{T}_j$ describing the orientation of the j th local frame with respect to the global coordinates can be represented in the new form

$$\hat{T}_j = [\hat{\underline{a}}_{jk}, \hat{\underline{S}}_j \times \hat{\underline{a}}_{jk}, \hat{\underline{S}}_j] . \quad (3.47)$$

Also, the direction cosines of $\hat{\underline{S}}_j$ and $\hat{\underline{a}}_{jk}$ can be obtained recursively from the initial direction cosines

$$\hat{\underline{S}}_1 = (0, 0, 1) \quad (3.48)$$

$$\hat{\underline{a}}_{12} = (C(\theta_1 + \beta_1), S(\theta_1 + \beta_1), 0) \quad (3.49)$$

and the following expressions

$$\hat{\underline{S}}_j = \hat{T}_{j-1} \begin{bmatrix} 0 \\ -S^{\alpha_{(j-1)j}} \\ C^{\alpha_{(j-1)j}} \end{bmatrix} \quad (3.50)$$

$$\hat{\underline{a}}_{jk} = \hat{T}_{j-1} \begin{bmatrix} C(\theta_j + \beta_j) \\ C^{\alpha_{(j-1)j}} S(\theta_j + \beta_j) \\ S^{\alpha_{(j-1)j}} S(\theta_j + \beta_j) \end{bmatrix}. \quad (3.51)$$

Notice that α_{jk} is the angle of twist about the link axis $\hat{\underline{a}}_{jk}$. The rate of change of orientation of link jk about the global coordinates is given by the sum of the relative angular velocities about the revolute joint axes, which can be written in terms of the first order rotational coefficients in the form

$$\underline{\dot{\omega}}_{jk} = [\hat{G}_{jk}] \underline{\dot{\phi}} = [\hat{G}_{jk}] \underline{\dot{\beta}}. \quad (3.52)$$

Recall that the arm is in a stationary configuration and parameters ϕ_n are invariant with respect to time. The first order geometric coefficients $[\hat{G}_{jk}]$ are distinct from those defined by the matrix $[G_{jk}]$, since their column vectors are given in terms of the deflected joint axes as follows

$$[\hat{G}_{jk}]_n = \begin{cases} \hat{\underline{S}}_n & , \quad n \leq j \quad , \quad n\text{-revolute} \\ 0 & \text{otherwise} . \end{cases} \quad (3.53)$$

The deflection position of the mass center C_j in the global reference frame is described by the equation

$$\hat{\underline{C}}_j = \hat{\underline{R}}_j + [\hat{T}_j] \underline{C}_j^{(j)} \quad (3.54)$$

where, $\underline{C}_j^{(j)}$ defines the position of C_j in the j th local frame, and vector $\hat{\underline{R}}_j$ locates the origin of this frame in the global coordinate system. From Fig. 3.4, $\underline{R}_j$ can be obtained as

$$\hat{\underline{R}}_j = s_{11} \hat{\underline{S}}_1 + \sum_{\ell=2}^j (\hat{a}_{(\ell-1)\ell} \hat{\underline{a}}_{(\ell-1)\ell} + s_{\ell} \hat{\underline{S}}_{\ell}). \quad (3.55)$$

Following the procedure described in Section 2.2, we can obtain the rate of change of mass center position

$$\dot{\hat{\underline{C}}}_j = [\hat{G}_{cj}] \dot{\hat{\underline{\phi}}} = [\hat{G}_{cj}] \dot{\hat{\underline{\rho}}}. \quad (3.56)$$

Matrix $[\hat{G}_{cj}]$ represents the first order translational influence coefficients of the deflected arm. The n th component of $[\hat{G}_{cj}]$ is defined as

$$[\hat{G}_{cj}]_n = \begin{cases} \hat{\underline{S}}_n \times (\hat{\underline{p}} - \hat{\underline{R}}_n) & , \quad n \leq j \quad ; \quad n\text{-revolute} \\ \hat{\underline{S}}_n & , \quad n \leq j \quad ; \quad n\text{-prismatic} \\ 0 & , \quad \text{otherwise} \end{cases} \quad (3.57)$$

Next, the total kinetic energy of the manipulator arm is obtained from Eqn. (2.43)

$$KE = \frac{1}{2} \dot{\underline{\beta}}^T [\hat{I}^*] \dot{\underline{\beta}} \quad (3.58)$$

where $[\hat{I}^*]$ represents the inertia matrix of the manipulator in its deflected configuration. The scalar component in row m , column n of this matrix is given by the equation

$$[\hat{I}^*]_{m;n} = \sum_{j=1}^N (M_j [\hat{G}_{cj}]_m^T [\hat{G}_{cj}]_n + [\hat{G}_{jk}]_m^T [\hat{\Pi}_{jk}] [\hat{G}_{jk}]_n). \quad (3.59)$$

Matrix $[\hat{\Pi}_{jk}^{(j)}]$ represents the inertia tensor about the centroid for link jk in terms of the global coordinate system. This matrix is defined by the equation (as shown in Chapter II)

$$[\hat{\Pi}_{jk}] = \hat{T}_j [\hat{\Pi}_{jk}^{(j)}] \hat{T}_j^T. \quad (3.60)$$

The 3×3 matrix $[\hat{\Pi}_{jk}^{(j)}]$ denotes the inertia tensor of link jk in terms of the j th reference frame.

The potential and dissipation energies of the mechanism are not defined in terms of the geometric

coefficients; therefore their mathematical definitions remain the same as those given by Eqns. (2.45) and (3.8). Now, substituting the final expressions for the kinetic, potential, and dissipation energies into Lagrange's equations of motion, we can write

$$\frac{d}{dt}([\hat{I}^*]_{n;\underline{\beta}}) - \frac{1}{2} \underline{\beta}^T \left(\frac{\partial}{\partial \underline{\beta}_n} [\hat{I}^*] \right) \underline{\beta} + [C^*]_{n;\underline{\beta}} + [K^*]_{n;\underline{\beta}} = \hat{T}_n. \quad (3.61)$$

Parameter $\hat{T}_n$ is the nth component of the vector $\hat{\underline{T}}$ representing the generalized forces of the arm. This vector is defined by the equation

$$\hat{\underline{T}} = \sum_{j=1}^N \hat{\underline{T}}^{(jk)} \quad (3.62)$$

where

$$\hat{\underline{T}}^{(jk)} = \underline{f}_p^T [\hat{G}_p] + \underline{m}_{jk}^T [\hat{G}_{jk}] \quad (3.63)$$

The first term on the left side of Eqn. (3.61) can be expanded as follows

$$\frac{d}{dt}([\hat{I}^*]_{n;\underline{\beta}}) = [\hat{I}^*]_{n;\underline{\beta}} + \left(\frac{d}{dt} [\hat{I}^*]_{n;} \right) \underline{\beta} \quad (3.64)$$

where

$$\frac{d}{dt} ([\hat{I}^*]_{n;}) = \sum_{m=1}^N \dot{\beta}_m \frac{\partial}{\partial \beta_m} ([\hat{I}^*]_{n;}) = \dot{\underline{\beta}}^T \frac{\partial}{\partial \underline{\beta}} [\hat{I}^*]_{n;}. \quad (3.65)$$

Notice that $\frac{\partial}{\partial \underline{\beta}} [\hat{I}^*]_{n;}$ represent the standard convention

differentiation with respect to a vector. The result of this differentiation is an $N \times N$ matrix, where the m th row of this matrix is obtained from the derivative of $[\hat{I}^*]_{n;}$ with respect to the m th component of vector $\underline{\beta}$, or

$$\frac{\partial}{\partial \underline{\beta}} ([\hat{I}^*]_{n;}) = \begin{bmatrix} \frac{\partial}{\partial \beta_1} [\hat{I}^*]_{n;}; \\ \frac{\partial}{\partial \beta_2} [\hat{I}^*]_{n;}; \\ \vdots \\ \frac{\partial}{\partial \beta_N} [\hat{I}^*]_{n;}; \end{bmatrix}. \quad (3.66)$$

Using Eqns. (3.64) and (3.65) in (3.61) gives the n th equation of motion as

$$[\hat{I}^*]_{n;\underline{\beta}} \ddot{\underline{\beta}} + \dot{\underline{\beta}}^T \left[\left(\frac{\partial}{\partial \underline{\beta}} [\hat{I}^*]_{n;}) - \frac{1}{2} \frac{\partial}{\partial \beta_n} ([\hat{I}^*]) \right) \right] \dot{\underline{\beta}} + [C^*]_{n;\underline{\beta}} \dot{\underline{\beta}} + [K^*]_{n;\underline{\beta}} = \hat{T}_n. \quad (3.67)$$

This equation can be rewritten in the form

$$[\hat{I}^*]_{n;\underline{\beta}}'' + \underline{\beta}^T [\hat{P}_n] \underline{\dot{\beta}} + [C^*]_{n;\underline{\beta}} \underline{\dot{\beta}} + [K^*]_{n;\underline{\beta}} \underline{\beta} = \hat{T}_n \quad (3.68)$$

where the inertia power matrix, $[\hat{P}_n]$, is defined by the equation

$$[\hat{P}_n] = \frac{\partial}{\partial \underline{\beta}} ([\hat{I}^*]_{n;}) - \frac{1}{2} \frac{\partial}{\partial \underline{\beta}_n} ([\hat{I}^*]) . \quad (3.69)$$

An alternative matrix, $[\hat{P}_n^*]$, is presented in [29] which can replace $[P_n]$ (this is shown in Appendix B), such that Eqn. (3.68) becomes

$$[\hat{I}^*]_{n;\underline{\beta}}'' + \underline{\beta}^T [\hat{P}_n^*] \underline{\dot{\beta}} + [C^*]_{n;\underline{\beta}} \underline{\dot{\beta}} + [K^*]_{n;\underline{\beta}} \underline{\beta} = \hat{T}_n . \quad (3.70)$$

The scalar component of $[\hat{P}_n^*]$ associated with the l th row and m th column is defined by the equation

$$\begin{aligned} [\hat{P}_n^*]_{l;m} = & \sum_{j=1}^N M_j [\hat{H}_{cj}]_{l;m}^T [\hat{G}_{cj}]_n + [\hat{H}_{jk}]_{l;m}^T [\hat{\Pi}_{jk}] [\hat{G}_{jk}]_n \\ & + [\hat{G}_{jk}]_l^T [\hat{\Pi}_{jk}] [\hat{G}_{jk}]_n [\hat{G}_{jk}]_m . \end{aligned} \quad (3.71)$$

The second order geometric influence coefficients are defined by $[\hat{H}_{jk}]$ and $[\hat{H}_{cj}]$, the expressions for these

coefficients are shown in Appendix A. The 3×3 matrix $[\hat{G}_{jk}]_n$ is constructed from the scalar components of the vector $[\hat{G}_{jk}]_n$ as follows

$$[\hat{G}_{jk}]_n = \begin{bmatrix} 0 & -[\hat{G}_{jk}^z]_n & [\hat{G}_{jk}^y]_n \\ [\hat{G}_{jk}^z]_n & 0 & -[\hat{G}_{jk}^x]_n \\ -[\hat{G}_{jk}^y]_n & [\hat{G}_{jk}^x]_n & 0 \end{bmatrix} \quad (3.72)$$

This matrix is introduced in the above form so that a vector cross product can be expressed in terms of matrix multiplication

$$[\hat{G}_{jk}]_n \times [\hat{G}_{jk}]_m = [\hat{G}_{jk}]_n [\hat{G}_{jk}]_m .$$

Recognizing that displacements β_n are small, Appendix C shows that the inertia and inertia power of the manipulator arm in its deflected configuration can be approximated as

$$[\hat{I}^*] \cong [I^*] \quad (3.74)$$

$$[\hat{P}_n^*] \cong [P_n^*] \quad (3.75)$$

where matrices $[I^*]$ and $[P_n^*]$ are the inertia and inertia power of the manipulator obtained from the

undeflected configuration of the arm. It is also shown in the Appendix C that

$$\hat{T}_n \cong T_n . \quad (3.76)$$

Considering the above approximations, Eqn. (3.70) can be simplified as

$$[I^*]_{n;\underline{\beta}} \ddot{\underline{\beta}} + \underline{\beta}^T [P_n^*] \dot{\underline{\beta}} + [C^*]_{n;\underline{\beta}} \dot{\underline{\beta}} + [K^*]_{n;\underline{\beta}} \underline{\beta} = T_n . \quad (3.77)$$

This equation can be solved numerically to obtain the system response.

3.6 Order Analysis

In order to predict the effect of the nonlinear terms on the system response, it is necessary to determine the significance of these terms in the equations of motion. This can be accomplished by estimating the order of the nonlinear terms relative to those of the linear terms. To simplify the procedure, we consider a six degree of freedom serial manipulator arm with flexible joints only. Hence, Eqn. (3.77) can be written in the form

$$\begin{aligned} \sum_{i=1}^N [I^*]_{n;i} \ddot{\beta}_i + \sum_{i=1}^N \sum_{j=1}^N [P_n^*]_{i;j} \dot{\beta}_i \dot{\beta}_j + \\ \sum_{i=1}^N [C^*]_{n;i} \dot{\beta}_i + K_n \beta_n = T_n . \end{aligned} \quad (3.78)$$

By comparing the definitions for system inertia and inertia power, it is evident that the magnitudes of their scalar components are of the same order, so it may be assumed that

$$| [I^*]_{\ell; m} | \cong | [P_n^*]_{\ell; m} | . \quad (3.79)$$

Furthermore, let us assume that the damping matrix is proportional to the stiffness matrix only, or

$$[C^*] = \nu [K^*] = 10^{-d} [K^*] \quad (3.80)$$

where d is a constant. Now, Eqn. (3.78) can be reduced to

$$\sum_{i=1}^N [I^*]_{n; i} \ddot{\beta}_i + \sum_{i=1}^N \sum_{j=1}^N [I^*]_{i; j} \dot{\beta}_i \dot{\beta}_j + 10^{-d} K_n \dot{\beta}_n + K_n \beta_n = T_n. \quad (3.81)$$

Since β_n represents the oscillation of joint n about its axis, it can be approximated by a periodic motion consisting of N harmonic functions. Considering only one mode of vibration at a time, we can write

$$| \beta_n | = B_n \quad (3.82)$$

$$| \dot{\beta}_n | = B_n \omega \quad (3.83)$$

$$| \ddot{\beta}_n | = B_n \omega^2 . \quad (3.84)$$

Parameter B_n defines the magnitude of the vibratory motion. To further simplify the equations of motion, assume that the magnitude of all joint displacements is equal to a maximum displacement B , or

$$B_n \cong B_m \cong B. \quad (3.85)$$

Introducing Eqns. (3.82) to (3.85) in (3.81), we obtain

$$B \left\{ -\omega^2 \sum_{i=1}^N [I^*]_{n;i} + B\omega^2 \sum_{i=1}^N \sum_{j=1}^N [I^*]_{i;j} + 10^{-d}\omega K_n + K_n \right\} = T_n. \quad (3.86)$$

The order of the scalar components associated with the n th row of $[I^*]$ are greater than those corresponding to the $(n+1)$ th row of this matrix. It is then possible to assume a worst condition for the inertia matrix, which is

$$\sum_{i=1}^N [I^*]_{n;i} \cong \sum_{i=1}^N [I^*]_{(n+1);i} \quad (3.87)$$

or

$$\sum_{i=1}^N \sum_{j=1}^N [I^*]_{i;j} \cong N \sum_{i=1}^N [I^*]_{n;i}. \quad (3.88)$$

Substituting Eqn. (3.88) into (3.86) results in

$$B\omega^2 \left(\sum_{i=1}^N [I^*]_{n;i} \right) (-1 + NB) + BK_n (10^{-d}\omega + 1) = T_n . \quad (3.89)$$

The product NB represents the significance of the nonlinear terms in the equations of motion. Assuming that the vibrational deformations are less than $\pm .02$ radians (a large motion for a precision manipulator), we can conclude that

$$| \underline{\dot{\hat{\theta}}}^T [P_n^*] \underline{\dot{\hat{\theta}}} | \leq 0.12 | [I^*]_{n;\underline{\hat{\theta}}}'' | . \quad (3.90)$$

Notice that the significance of the damping factor depends on the value of ω .

All of the proposed approximations correspond to worst conditions of the arm; therefore, the significance of the nonlinear terms is grossly exaggerated. By making more realistic assumptions it is possible to draw the same conclusion for a manipulator arm with a large number of freedoms.

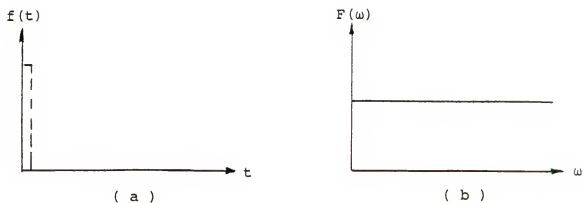


Figure 3.1. Fourier Transform of Unit Impulse.

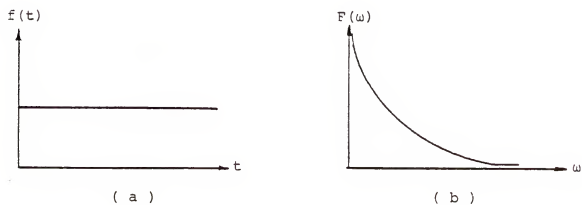
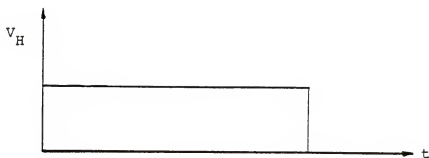
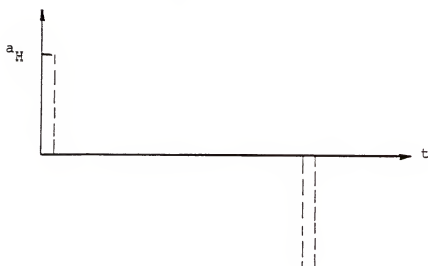


Figure 3.2. Fourier Transform of Unit Step.



(a)



(b)

Figure 3.3. Constant Velocity Curve.

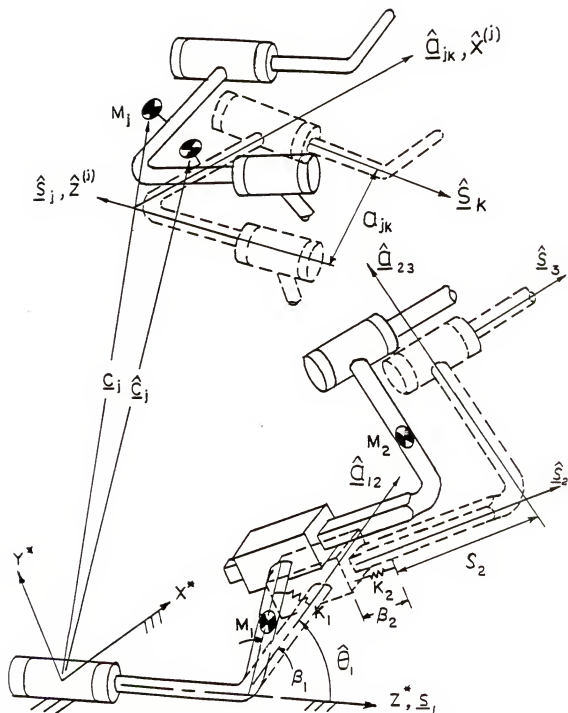


Figure 3.4. Kinematic Representation of Oscillating Manipulator.

CHAPTER IV

ARM DEFLECTION DUE TO THE GROSS MOTION

In all previous considerations in the vibration analysis of flexible manipulators, it is assumed that the manipulator arms are in a fixed configuration. As the operational speed increases, the inertia loads become significantly large. At lower speeds the frequency content of these forces are much lower than the arm's fundamental frequency. Under these circumstances, the inertia loads act as statically applied loads varying throughout the cycle. Thus, the system deformations can be obtained by application of quasi-static analysis. At certain speeds, the frequency content of inertia forces becomes so large that they may excite one or more modes of vibration of the mechanism. The manipulator's deformation will then become time-dependent displacements which are coupled with the mechanism's gross motion.

For precision control of a manipulator, a complete dynamic formulation can be used to predict, and therefore compensate for, the system deformations under load

compensate for, the system deformations under load (inertia and external loads). The objective is to obtain compensation signals which, when added to the control signals, correct for hand positioning errors due to the flexible components. Knowing the active loads and manipulator's dynamic parameters (local masses and stiffnesses), the change in the terminal device position, ΔP_H , can be calculated off-line, prior to operation. The compensation values are then obtained in order to modify the nominal actuator commands.

4.1 Complete Dynamic Model for Manipulator Arms

A general approach based on the formulation of Section 3.5 is described here for deriving the equations of motion of flexible serial manipulator arms. This approach considers the kinetic energy contribution of the system inertia produced by both nominal and perturbed motion of the arm. The perturbed coordinates are regarded as the unknowns, but the nominal motion of the arm is considered to be known. Hence, the equations of motion are expressed in terms of the relative perturbed displacements of the arm.

Consider the flexible manipulator arm of Fig. 4.1. The geometric parameters of this robot arm are defined in Section 3.5. However, the motion of each joint is described by the sum of two time dependent displacements

associated with the nominal motion and deformation of the joint. So, the relative motion of joint n can be defined by the equation

$$\hat{\phi}_n(t) = \phi_n(t) + \beta_n(t) . \quad (4.1)$$

The absolute angular velocity of link jk about the global coordinates is defined as a product of the corresponding first order rotational coefficients and the joint velocities

$$\hat{\omega}_{jk} = [\hat{G}_{jk}] \hat{\underline{\phi}} = [\hat{G}_{jk}] (\dot{\underline{\phi}} + \dot{\underline{\beta}}) . \quad (4.2)$$

It must be borne in mind that $\dot{\phi}_n$ is identically zero if joint n represents a pseudo-input. Furthermore, the translational velocity of mass center C_j can be described in terms of the joint velocities and translational influence coefficients

$$\hat{\underline{C}}_j = [\hat{G}_{Cj}] \hat{\underline{\phi}} = [\hat{G}_{Cj}] (\dot{\underline{\phi}} + \dot{\underline{\beta}}) . \quad (4.3)$$

Introducing Eqns. (4.2) and (4.3) in (2.37), the kinetic energy of this mechanism can be expressed in the form

$$KE = \frac{1}{2} (\dot{\underline{\phi}} + \dot{\underline{\beta}})^T [\hat{I}^*] (\dot{\underline{\phi}} + \dot{\underline{\beta}}) . \quad (4.4)$$

Recall that gross motion of the arm has no effect on the system's potential and dissipation energies, since the mechanism is assumed to vibrate about its instantaneous equilibrium configuration. The equations of motion can now be derived from Lagrange's equations

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{\beta}_n} \right) - \frac{\partial KE}{\partial \beta_n} + \frac{\partial W^D}{\partial \dot{\beta}_n} + \frac{\partial PE}{\partial \beta_n} = \hat{T}_n . \quad (4.5)$$

Nominal displacements, ϕ_n , do not represent the independent generalized coordinates of the system, since they are a set of given parameters which describe the time varying characteristics of the manipulator arm. Therefore, only perturbed displacements are considered as generalized coordinates in the Lagrangian formulation. Now, substitute the expressions for the system kinetic, potential, and dissipation energies into Eqn. (4.5) to obtain

$$\begin{aligned} \frac{d}{dt} \{ [\hat{I}^*]_n; (\dot{\underline{\phi}} + \dot{\underline{\beta}}) \} - \frac{1}{2} (\dot{\underline{\phi}} + \dot{\underline{\beta}})^T \left(\frac{\partial}{\partial \beta_n} [I^*] \right) (\dot{\underline{\phi}} + \dot{\underline{\beta}}) \\ + [C^*]_n; \dot{\underline{\beta}} + [K^*]_n; \underline{\beta} = \hat{T}_n . \end{aligned} \quad (4.6)$$

The first term on the left hand side of Eqn. (4.6) can be expanded as follows

$$\begin{aligned} \frac{d}{dt} \{ [\hat{I}^*]_n; (\dot{\underline{\phi}} + \dot{\underline{\beta}}) \} = [\hat{I}^*]_n; (\ddot{\underline{\phi}} + \ddot{\underline{\beta}}) + \\ (\dot{\underline{\phi}} + \dot{\underline{\beta}})^T \left(\frac{\partial}{\partial \underline{\beta}} [I^*]_n; \right) (\dot{\underline{\phi}} + \dot{\underline{\beta}}) . \end{aligned} \quad (4.7)$$

Introducing Eqn. (4.7) in (4.6) results in

$$[I^*]_n;(\ddot{\underline{\phi}} + \ddot{\underline{\beta}}) + (\dot{\underline{\phi}} + \dot{\underline{\beta}})^T [\hat{P}_n^*] (\dot{\underline{\phi}} + \dot{\underline{\beta}}) + [C^*]_n; \dot{\underline{\beta}} + [K^*]_n; \underline{\beta} = \hat{T}_n . \quad (4.8)$$

The scalar components of $[\hat{P}_n^*]$ are defined by Eqn. (3.71). Expanding Eqn. (4.8) and rearranging the resultant equation gives a nonlinear differential equation in terms of the perturbed coordinates β_n ,

$$[\hat{I}^*]_n; \ddot{\underline{\beta}} + \dot{\underline{\beta}}^T [\hat{P}_n^*] \dot{\underline{\beta}} + \dot{\underline{\phi}}^T [\hat{R}_n^*] \dot{\underline{\beta}} + [C^*]_n; \dot{\underline{\beta}} + [K^*]_n; \underline{\beta} = \hat{T}_n - [\hat{I}^*]_n; \ddot{\underline{\phi}} - \dot{\underline{\phi}}^T [\hat{P}_n^*] \dot{\underline{\phi}} . \quad (4.9)$$

Notice that the first time derivatives of the nominal and perturbed coordinates are coupled in this equation. The coupling matrix $[\hat{R}_n^*]$ is an $N \times N$ matrix defined by the equation

$$[\hat{R}_n^*] = [\hat{P}_n^*]^T + [\hat{P}_n^*] . \quad (4.10)$$

Matrices $[\hat{I}^*]$, $[\hat{P}_n^*]$, and $[\hat{R}_n^*]$ are functions of the deformation displacements β_n . However, recalling Eqns. (3.74) to (3.76), the coefficients of differential equations of motion can be obtained from the nominal displacements only. Thus, Eqn. (4.9) can be rewritten in the form

$$[I^*]_{n;\underline{\underline{\beta}}} \ddot{\underline{\underline{\beta}}} + \underline{\underline{\beta}}^T [P_n^*] \dot{\underline{\underline{\beta}}} + \underline{\underline{\phi}}^T [R_n^*] \dot{\underline{\underline{\beta}}} + [C^*]_{n;\underline{\underline{\beta}}} \dot{\underline{\underline{\beta}}} + [K^*]_{n;\underline{\underline{\beta}}} \underline{\underline{\beta}} = T_n - [I^*]_{n;\underline{\underline{\phi}}} \ddot{\underline{\underline{\phi}}} - \underline{\underline{\phi}}^T [P_n^*] \dot{\underline{\underline{\phi}}} . \quad (4.11)$$

The last two terms on the right hand side of Eqn. (4.11) represent the equivalent load acting over the nth input due to the nominal motion of the system inertia. This load acts as an externally applied load on the system which causes the arm to deform about its rigid-body position.

In order to obtain the perturbed motion of the arm, Eqn. (4.11) must be integrated numerically. Deformation displacements can be computed at a series of time steps Δt , to describe the position of the arm at any time t . The accuracy of any numerical integration depends on the size of time steps, Δt , which becomes even more significant for Eqn. (4.11) since the coefficients of this differential equation are time varying functions. To minimize the cumulative errors in step-by-step integration and avoid numerical instability, numerical integration must be performed at small time steps Δt . An adequately small time step can be selected as a fraction of the time period associated with the maximum excited mode of vibration. For example, if only the first mode of the arm is excited by an applied load, the time step should be a fraction of $1/\omega_1$. Clearly small time steps require long computational times for numerical integration. The

computational time may be reduced to some extent by using a suitable method such as fourth-order Runge-Kutta method combined with Haming predictor corrector [28]. There are several other numerical methods [30] which can be used for solving a dynamic equation such as Eqn. (4.11).

As mentioned previously, if the frequency content of the inertia loads is significantly lower than the fundamental frequency, the variation of deformation displacements with respect to time can be neglected. Therefore, Eqn. (4.11) can be readily simplified by neglecting the time derivatives of perturbed coordinates β_n , or

$$[K^*]_n \underline{\beta} = T_n - [I^*]_n \ddot{\underline{\phi}} - \dot{\underline{\phi}}^T [P_n^*] \dot{\underline{\phi}}. \quad (4.12)$$

This equation represents the quasi-static approximation of system deformations. In most cases, such analysis is adequate for describing the manipulator's deflection due to inertia loads.

4.2 Variation of System Natural Frequencies Due to the Nominal Motion of the Arm

The effect of manipulator's gross motion on the natural frequencies may best be illustrated by an example problem. Figure 4.2 shows the example manipulator and the

kinematic notation describing it. The manipulator is an open loop spatial RR-R-RRR mechanism, where R stands for revolute joint and R-R-R indicates that the three successive joint axes are parallel. The joint axes are described by vectors $\underline{S}_j$, and $\underline{a}_{jk}$ is the unit vector along the common perpendicular between $\underline{S}_j$ and $\underline{S}_k$. The nominal motion of each joint is represented by the angle θ_j rotating about $\underline{S}_j$ axis (this angle is measured in a right-hand sense between $\underline{a}_{ij}$ and $\underline{a}_{jk}$). Parameters S_{jj} and a_{jk} represent the offset distance between links $\underline{a}_{ij}$ and $\underline{a}_{jk}$, and the link length between axes $\underline{S}_j$ and $\underline{S}_k$, respectively. The angle of twist between $\underline{S}_j$ and $\underline{S}_k$ is designated by angle α_{jk} . The geometric parameters for this robot arm are shown in Table 4.1. In order to represent an illustrative formulation, only one flexible component at a time is considered. Figure 4.2 shows that the manipulator arm deflects about joint axis $\underline{S}_1$ by an angle β_1 . The flexibility of the joint is represented by a torsional spring K_1 . Therefore, this system has one deformational degree of freedom only. Figure 4.3 illustrates the instantaneous position of the arm which lies in the $X^* - Z^*$ plane. The direction cosines of the joint and link axes for arm position are listed in Table 4.2. Furthermore, the associated mass of link jk is lumped at a point C_j (Fig. 4.3) on the link, the mass moments of inertia corresponding to the link are neglected in order to avoid computational complications.

The homogeneous equations of motion for this robot arm can be obtained from Eqn. (4.11). Recalling that the nonlinear terms, $\dot{\underline{p}}^T [P_n^*] \dot{\underline{p}}$, are insignificant and ignoring the damping forces, we can write

$$[I^*]_{1,1} \ddot{\beta}_1 + (\dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 \dot{\theta}_4 \dot{\theta}_5 \dot{\theta}_6) [R_1^*] \begin{bmatrix} \dot{\beta}_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + K_1 \beta_1 = 0. \quad (4.13)$$

Notice that Eqn. (4.13) describes the motion of a one degree of freedom system. Assuming that the nominal angular velocities of all joints are equal to a maximum frequency, Ω , Eqn. (4.13) is reduced to

$$[I^*]_{1,1} \ddot{\beta}_1 + \Omega Q_1^* \dot{\beta}_1 + K_1 \beta_1 = 0. \quad (4.14)$$

where

$$Q_1^* = \sum_{i=1}^6 [R_1^*]_{i,1}. \quad (4.15)$$

It is necessary to obtain the first and second order translational coefficients for mass centers C_j in order to evaluate $[I^*]_{1,1}$ and Q_1^* . These coefficients are listed in Tables 4.3 to 4.9. Rotational geometric coefficients are not needed since the mass moments of inertia have been neglected. Now, using Eqn. (2.44), we obtain

$$[I^*]_{1;1} = 1600M_2 + 6403M_3 + 8103M_4 + 8103M_5 + 12104M_6 . \quad (4.16)$$

Also, from Eqns. (4.10) and (3.71) we can evaluate the scalar components of $[R_1^*]$

$$[R_1^*]_{1;1} = 0 \quad (4.17)$$

$$[R_1^*]_{2;1} = -2400M_2 - 11.2M_3 - 16M_4 - 16M_5 - 27M_6 \quad (4.18)$$

$$[R_1^*]_{3;1} = 4794M_3 + 5390M_4 + 5390M_5 + 6580M_6 \quad (4.19)$$

$$[R_1^*]_{4;1} = -3.2M_4 - 3.2M_5 - 12M_6 \quad (4.20)$$

$$[R_1^*]_{5;1} = [R_1^*]_{6;1} = 0 \quad (4.21)$$

from which the value of Q_1^* is determined.

$$Q_1^* = -2400M_2 + 4782.8M_3 + 5370M_4 + 5370M_5 + 6541M_6 \quad (4.22)$$

Comparing Eqns. (4.16) and (4.22), it is evident that the value of Q_1^* is much smaller than $[I^*]_{1;1}$.

Therefore, we can consider the worst value of Q_1^* which is

$$Q_1^* \approx [I^*]_{1;1} \quad (4.23)$$

The equation of motion can now be rewritten in the form

$$\ddot{\beta}_1 + \Omega \dot{\beta}_1 + \omega_1^2 \beta_1 = 0 \quad (4.24)$$

where ω_1 is the natural frequency of the arm corresponding to its fixed configuration and it is defined by the equation

$$\omega_1 = \sqrt{\frac{K_1}{[I^*]_{1;1}}} \quad (4.25)$$

The nominal angular velocity of each joint can be defined as

$$\Omega = \epsilon \omega_1 \quad (4.26)$$

This equation implies that Ω is proportional to the natural frequency by a factor ϵ . The eigenvalue of Eqn. (4.24) is given by the equation

$$\omega_{d1} = \sqrt{\omega_1^2 - \Omega^2} = \omega_1^2 \sqrt{1 - \epsilon^2} \quad (4.27)$$

Thus, when the arm is operating with input velocities of $\epsilon \omega_1$, its natural frequency will drop from ω_1 to ω_{d1} . Selecting an exaggerated value of 0.1 for ϵ results in

$$\omega_{d1} = 0.995 \omega_1 \quad (4.28)$$

It is apparent from this equation that the effect of the manipulator's gross motion on the system natural frequency is insignificant. This procedure can be repeated considering any or all other flexible components for any configuration of the arm to arrive at the same conclusion.

4.3 Compensation for Deformations

To compensate for the manipulator's deformations, the overall change in the spatial position of the hand can be evaluated from the known deflection in each flexible component. On the basis of this known positional error, corrections can be made to the input command signals which move the hand to its desired position. This procedure to enhance the positioning accuracy of the hand can be said to make the manipulator "electronically rigid". In this way, system inputs are tuned to the operating conditions to enhance positioning precision. The concept of compensation for deformations by tuning the system inputs is presented in [2] for high-speed cam systems.

The overall change in the spatial position of the end-effector can be represented by a vector, $\Delta \underline{P}_H$, consisting of six components. This vector describes the deflection of a point on the end-effector by its first three components and the change in the orientation of the

hand by its last three components. Vector, $\Delta \underline{P}_H$, can be obtained from the equation

$$\Delta \underline{P}_H = \left[\frac{\underline{G}_H}{\underline{G}_{N(N+1)}} \right] \underline{\beta} = [\underline{J}] \underline{\beta} \quad (4.29)$$

where $[\underline{J}]$ is known as the Jacobian matrix. Recall that β_N represents the deflections in the manipulator's inputs and pseudo-inputs. However, only the actual inputs of the arm can be tuned to correct for the error in the position of the end-effector. Therefore, it is necessary to predict the required change in the nominal motion of the arm to produce a deflection, $\Delta \underline{P}_H$, in the spatial position of manipulator's hand. Considering the nominal motion of the arm, we can write

$$\Delta \underline{P}_H = \left[\frac{\underline{R}_{G_H}}{\underline{R}_{G_{N(N+1)}}} \right] \Delta \underline{\phi} = [\underline{R}\underline{J}] \Delta \underline{\phi} \quad (4.30)$$

Matrix $[\underline{R}\underline{J}]$ represents the Jacobian of the rigid-body mechanism and vector $\Delta \underline{\phi}$ is the required correction vector. For a six degree of freedom manipulator arm, the correction values can be obtained from

$$\Delta \underline{\phi} = [\underline{R}\underline{J}]^{-1} \Delta \underline{P}_H \quad (4.31)$$

This should give good results when the deformations are small and the Jacobian matrix, $[RJ]$, is not near a singularity. For manipulators with redundant degrees of freedom, the pseudo-inverse of the Jacobian can be used, or

$$\Delta \underline{\phi} = ([RJ]^T [RJ])^{-1} [RJ]^T \Delta \underline{P}_H \quad (4.32)$$

Next, the tuned values for the system inputs, $T_{\underline{\phi}}$, can be obtained from

$$T_{\underline{\phi}} = \underline{\phi} - \Delta \underline{\phi}. \quad (4.33)$$

This is a relatively simple type of compensation which shows that, it is possible to electronically correct for flexibility in a manipulator system if the system loads and dynamic parameters are known.

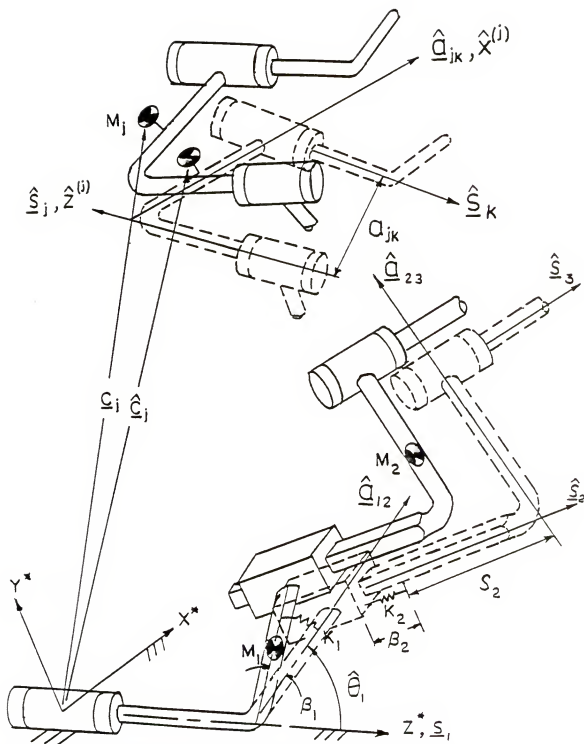


Figure 4.1. Kinematic Representation of Oscillating Manipulator.

$\otimes$ - vector directed into the page

$\odot$ - vector directed out of the page

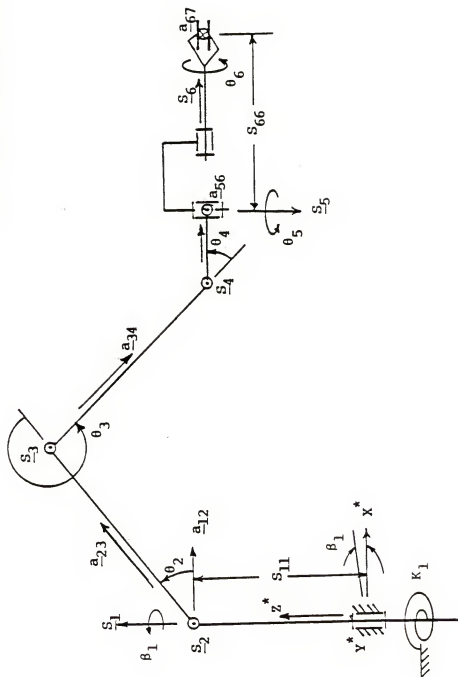


Figure 4.2. Kinematic Representation of the Example Manipulator.

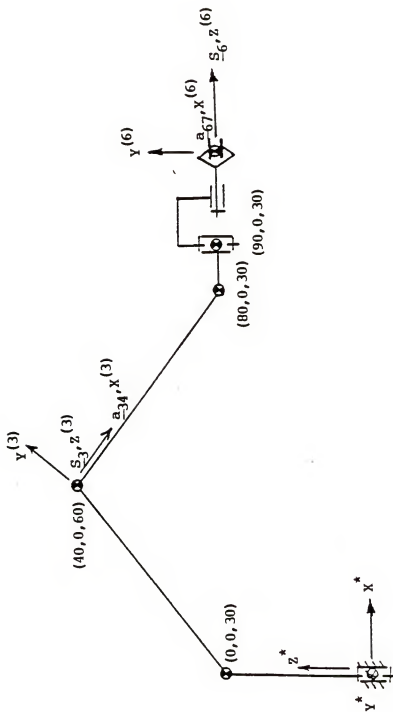


Figure 4.3. Arm Position for the Example Analysis.

Table 4.1. Geometrical Parameters for the Example Manipulator.

j	S_{jj} (in.)	a_{jk} (in.)	α_{jk} (deg.)
1	30	0	90
2	0	50	0
3	0	50	0
4	0	10	90
5	0	0	90
6	20	0	--

Table 4.2. Manipulator Position for the Example Analysis.

j	$\underline{S}_j$	$\underline{a}_{jk}$	$\underline{R}_{jk}$ (in.)	θ_j (deg.)
1	(0,0,1)	(1,0,0,)	(0,0,30)	0.0
2	(0,-1,0)	(.8,0,.6)	(0,0,30,)	36.9
3	(0,-1,0)	(.8,0,-.6)	(40,0,60)	-73.7
4	(0,-1,0,)	(1,0,0)	(80,0,30)	36.9
5	(0,0,-1)	(0,-1,0)	(90,0,30)	90.0
6	(1,0,0)	(0,1,0)	(110,0,30)	180.0

Table 4.3. First Order Translational Influence Coefficients
for Mass Centers C_j

$$[G_{C1}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_{C2}] = \begin{bmatrix} 0 & -30 & 0 & 0 & 0 & 0 \\ 40 & 0 & 0 & 0 & 0 & 0 \\ 0 & 40 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_{C3}] = \begin{bmatrix} 0 & 0 & 30 & 0 & 0 & 0 \\ 80 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 40 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_{C4}] = \begin{bmatrix} 0 & 0 & 30 & 0 & 0 & 0 \\ 90 & 0 & 0 & 0 & 0 & 0 \\ 0 & 90 & 50 & 10 & 0 & 0 \end{bmatrix}$$

$$[G_{C5}] = \begin{bmatrix} 0 & 0 & 30 & 0 & 0 & 0 \\ 90 & 0 & 0 & 0 & 0 & 0 \\ 0 & 90 & 50 & 10 & 0 & 0 \end{bmatrix}$$

$$[G_{C6}] = \begin{bmatrix} 0 & 0 & 30 & 0 & 0 & 0 \\ 110 & 0 & 0 & 0 & -20 & 0 \\ 0 & 110 & 70 & 30 & 0 & 0 \end{bmatrix}$$

Table 4.6. Second Order Translational Influence Coefficients
for Mass Center C_3

$$[H_{C3}] =$$

-80	0	0	0	0	0	0
0	0	30	0	0	0	0
0	0	0	0	0	0	0
	-80	-40	0	0	0	0
	0	0	0	0	0	0
	0	30	0	0	0	0
		-40	0	0	0	0
		0	0	0	0	0
		30	0	0	0	0
			0	0	0	0
			0	0	0	0
			0	0	0	0
				0	0	0
				0	0	0
				0	0	0
					0	0
					0	0
					0	0
						0
						0
						0

Symmetric

Table 4.8. Second Order Translational Influence Coefficients
for Mass Center C_5

$$[H_{C5}] =$$

-90	0	0	0	0	0	0
0	0	30	0	0	0	0
0	0	0	0	0	0	0
	-90	-50	-10	0	0	0
	0	0	0	0	0	0
	0	30	0	0	0	0
		-50	-10	0	0	0
		0	0	0	0	0
		30	0	0	0	0
			-10	0	0	0
			0	0	0	0
			0	0	0	0
				0	0	0
				0	0	0
				0	0	0
					0	0
					0	0
					0	0

Symmetric

Table 4.9. Second Order Translational Influence Coefficients
for Mass Center C_6

$$[H_{C6}] =$$

-110	0	0	0	20	0
0	0	30	0	0	0
0	0	0	0	0	0
	-110	-70	-30	0	0
	0	0	0	0	0
	0	30	0	0	0
		-70	-30	0	0
		0	0	0	0
		30	0	0	0
			-30	0	0
			0	0	0
			0	0	0
				-20	0
				0	0
				0	0
					0
					0
					0

Symmetric

CHAPTER V

PARAMETRIC IDENTIFICATION

Establishing the dynamic model of a mechanical system such as a robot manipulator is an essential first step in creating a modern intelligent manufacturing system. To establish the complete dynamic model of a manipulator arm, it is necessary to evaluate the system's structural properties (i.e. mass, stiffness, and damping). The structural properties of a mechanical system are commonly determined from either the continuously distributed model of the system components, or the finite element methods using the mechanical properties of system elements. In general, manipulator link components are not uniform beams, therefore, the continuous model will not represent the system accurately. This problem is overcome by dividing the link elements into many small segments using finite element methods. Nevertheless, neither of the above methods are capable of providing a true representation of the actual mechanism. There are many factors including, the mass content of the actuators and drive trains, the deformational energy associated with the joint components, and the bearing flexibilities which

contribute significantly to the overall structural behavior of a manipulator arm. It is not possible to predict the effect of these factors on the structural behavior of the arm using the aforementioned methods. A reliable procedure is required to identify the system's dynamic parameters which considers all significant contributing factors. An experimental procedure is proposed to obtain system stiffness and mass parameters based on the modal analysis concept. This approach develops a true representation of the arm because the dynamic model parameters are found from the identification of the actual system.

5.1 Experimental Modal Analysis

Experimental modal analysis is a powerful tool for investigating the vibrational behavior of a flexible system. This tool has been widely used to describe the vibration characteristics of aircraft, spacecraft, automobile frames, machine tools, and other structures such as bridges. This chapter presents a technique for identifying the dynamic parameters of a robot manipulator utilizing experimental modal analysis. Before introducing this technique, experimental modal analysis is briefly discussed in order to provide a basic background for this concept.

Consider a system which can be described by the two degrees of freedom model shown in Fig. 5.1. The vibratory motion of this system is fully defined by the coordinates $x_1(t)$ and $x_2(t)$ which respectively represent the displacements of m_1 and m_2 from their static equilibrium position at an arbitrary time t . The system's differential equations of motion can be obtained from Lagrange's formulation

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{x}_n} \right) - \frac{\partial KE}{\partial x_n} + \frac{\partial W^D}{\partial \dot{x}_n} + \frac{\partial PE}{\partial x_n} = T_n . \quad (5.1)$$

The total kinetic energy of the system is defined by the equation

$$KE = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 . \quad (5.2)$$

The system's potential energy is due to the elongation of springs k_1 and k_2 , which can be described by the following equation

$$PE = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_1 - x_2)^2 . \quad (5.3)$$

Rayleigh's dissipation function is obtained from

$$W^D = \frac{1}{2} C_1 \dot{x}_1^2 + \frac{1}{2} C_2 (\dot{x}_1 - \dot{x}_2)^2 . \quad (5.4)$$

By taking the appropriate derivatives in Eqns. (5.2), (5.3), and (5.4), and inserting the results into Eqn. (5.1), we can write the equations of motion for this two degree of freedom system in the matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} . \quad (5.5)$$

T_1 and T_2 are the generalized forces acting over the generalized coordinates x_1 and x_2 , respectively. Assuming that the system is excited by a time-dependent forcing function $f_2(t)$ acting over the coordinate x_2 , the generalized forces T_1 and T_2 can be determined from the principle of virtual work

$$\delta W = f_2(t) \delta x_2 = T_1 \delta x_1 + T_2 \delta x_2 . \quad (5.6)$$

By inspection of this equation we conclude

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 0 \\ f_2(t) \end{bmatrix} . \quad (5.7)$$

The equations of motion can now be transformed into frequency domain using the Fourier transformations

$$F x_n(t) = x_n(\omega) \quad (5.8)$$

$$F \dot{x}_n(t) = i \omega x_n(\omega) \quad (5.9)$$

$$F \ddot{x}_n(t) = -\omega^2 x_n(\omega) \quad (5.10)$$

$$F f_2(t) = F_2(\omega) \quad . \quad (5.11)$$

Substituting Eqns. (5.7) through (5.11) into (5.5) gives

$$-\omega^2 [M] \underline{x}(\omega) + i \omega [C] \underline{x}(\omega) + [K] \underline{x}(\omega) = \begin{bmatrix} 0 \\ F_2(\omega) \end{bmatrix} \quad (5.12)$$

In order to decouple the equations of motion it is necessary to obtain the natural frequencies and the associated mode shapes of the system from the free vibration of the undamped system. The differential equations of motion for free vibration of the system reduce to

$$[M] \ddot{\underline{x}} + [K] \underline{x} = 0 \quad . \quad (5.13)$$

This leads to a standard eigenvalue problem described by the equation

$$[K]^{-1} [M] - \frac{1}{\omega^2} [I] = 0 \quad . \quad (5.14)$$

Next, the natural frequencies, ω_1 and ω_2 , along with the eigenvectors, $\underline{\psi}^{(1)}$ and $\underline{\psi}^{(2)}$, are obtained from Eqn. (5.14). Furthermore, the eigenvectors can be normalized so that

$$[\psi] = [\psi^{(1)} \quad \psi^{(2)}] = \begin{bmatrix} \psi_1^{(1)} & \psi_1^{(2)} \\ 1 & 1 \end{bmatrix} \quad (5.15)$$

Using the superposition theorem, the response can be described by a linear transformation of the $\underline{x}$ coordinates into the modal coordinates

$$\underline{x}(\omega) = [\psi] \underline{\eta}(\omega) \quad (5.16)$$

Assuming the damping matrix $[C]$ to be proportional to the stiffness matrix $[K]$, the equations of motion can be decoupled by introducing Eqn. (5.16) into (5.12) and premultiplying the resultant equation by $[\psi]^T$

$$-\omega^2 [\bar{M}] \underline{\eta}(\omega) + i\omega [\bar{C}] \underline{\eta}(\omega) + [\bar{K}] \underline{\eta}(\omega) = \underline{N}(\omega) \quad (5.17)$$

Matrices $[\bar{M}]$, $[\bar{C}]$, and $[\bar{K}]$ designate the modal parameters of the system, and $\underline{N}$ represents the generalized forces acting on the modal coordinates, which can be defined by the equation

$$\underline{N}(\omega) = [\psi]^T \begin{bmatrix} 0 \\ F_2(\omega) \end{bmatrix} = \begin{bmatrix} F_2(\omega) \\ F_2(\omega) \end{bmatrix} \quad (5.18)$$

The transfer functions in the modal coordinates can then be deduced from Eqn. (5.17) as shown below

$$h_{rr}(\omega) = \frac{\eta_r}{N_r} = \frac{\eta_r}{F_2} = \frac{1}{\bar{K}_r \left[1 - \left(\frac{\omega}{\omega_r} \right)^2 + 2i \zeta_r \left(\frac{\omega}{\omega_r} \right)^2 \right]} \quad (5.19)$$

where

$$\zeta_r = \frac{\bar{C}_r}{2(\bar{K}_r M_r)^{1/2}} \quad (5.20)$$

These transfer functions are complex functions which define the vibration characteristics of the system. The real parts of these functions are obtained from Eqn. (5.19)

$$\text{Re}(h_{rr}(\omega)) = \frac{1 - \left(\frac{\omega}{\omega_r} \right)^2}{\bar{K}_r \left[\left(1 - \left(\frac{\omega}{\omega_r} \right)^2 \right)^2 + 4\zeta_r^2 \left(\frac{\omega}{\omega_r} \right)^2 \right]} \quad (5.21)$$

Equation (5.21) can be graphically expressed as functions of ω as shown in Fig. 5.2. By inspection of Eqn. (5.21), it is evident that the magnitude of $\text{Re}(h_{rr}(\omega))$ is zero when ω is equal to ω_r . Furthermore, the maximum and minimum of $\text{Re}(h_{rr}(\omega))$ are obtained by differentiating Eqn. (5.21) with respect to ω and letting the result be zero. If this is done, we conclude that the maximum and minimum occur at

$$\omega = \omega_r (1 \pm \zeta_r) . \quad (5.22)$$

Substituting Eqn. (5.22) into (5.21) we can write

$$| \operatorname{Re}(h_{rr}(\omega)) |_{\max} + | \operatorname{Re}(h_{rr}(\omega)) |_{\min} = \frac{1}{2\bar{K}_r \zeta_r} . \quad (5.23)$$

The system transfer functions in the generalized coordinates can be obtained from Eqn. (5.16). The natural modes of the system are then obtained from these transfer functions

$$\underline{\psi}(r) = \begin{bmatrix} Q_{1r} \\ Q_{2r} \\ 1 \end{bmatrix} . \quad (5.24)$$

Parameters, Q_{1r} and Q_{2r} , represent the peak to peak values of the transfer functions as shown in Fig.

5.3. Therefore, the system characteristics are completely defined by the corresponding transfer functions.

Experimental modal analysis involves the extraction of resonance frequencies ω_r , modal parameters (damping, mass, and stiffness), and mode shape coefficients $\underline{\psi}(r)$ from the frequency response data $h_{nm}(\omega)$.

Modal extraction involves a number of curve fitting operations. A particular type of model is fit to the data and the best estimate of associated parameters is made.

These parameters can then be utilized to determine the dynamic parameters of the system from the equations

$$[M] = [\psi]^{-T} [\bar{M}] [\psi]^{-1} \quad (5.25)$$

$$[K] = [\psi]^{-T} [\bar{K}] [\psi]^{-1} \quad (5.26)$$

$$[C] = [\psi]^{-T} [\bar{C}] [\psi]^{-1} \quad (5.27)$$

Notice that the components of the mass, stiffness, and damping matrices are defined in terms of the local parameters of the system. Thus all dynamic parameters can be identified once the system matrices are calculated.

5.2 Parametric Identification for Manipulator Arms

This section presents a technique to identify the dynamic parameters of a particular industrial arm (Cincinnati T3-776 robot shown in Fig. 5.4) based on the concept described in Section 5.1. In order to simplify the identification procedure the system's degrees of freedom are minimized by excluding the motion of the manipulator's wrist and the deflection of the arm in the $X^* - Y^*$ plane. The compliance of the T3-776 robot associated with the servo drives is found to be insignificant relative to the structural compliance of the arm. This was determined by a comparison of the transfer functions measured with the servo brakes on and off.

Therefore, servo flexibilities are excluded by activating the servo brakes during the measurements. Although these limitations simplify the analysis substantially, they do not impose any restriction on the identification method.

To describe the natural motion of the T3-776 robot at a selected configuration, eleven points along the arm are chosen as shown in Fig. 5.5. The coordinates of these points in terms of the global reference frame are listed in Table 5.1. All the system modes are excited at an appropriate reference point by means of an instrumented impact hammer. The absolute vibrational response of each point is measured sequentially by an accelerometer. The excitation is obtained as an analog signal by a load cell mounted between the hammer's striking head and body. The analog signal is then transformed into the frequency domain by an A-D conversion followed by the Fast Fourier transformation of the resultant digital signal.

A preliminary set of measurements showed that the dominant vibrational modes of this system are concentrated in a frequency range of zero to sixty Hertz. Therefore, the frequency content of the exciting force is lowered to include only the frequency range of interest by enlarging the time duration of the impulsive force. The time duration can be increased by using a soft striking head during the measurements. Alternatively, the impulse duration can be further increased by the addition of mass

to the hammer body. The exciting force and its Fourier transform under the aforementioned conditions are shown in Fig. 5.6.

The vibrational motion of each point in the X^* and Z^* directions are measured by a large seismic accelerometer since the manipulator arm oscillates at low frequencies. The response of each point is then transformed from the time domain into frequency domain by a Fourier analyzer. An example time domain response of point 1 in the Z^* direction produced by an impulsive excitation is shown in Fig. 5.7.

The transfer functions of this system are obtained dividing the Fourier transform of the measured response by the Fourier transform of exciting force. Figure 5.8a represents the magnitude of the direct transfer function¹ associated with point 4 in the Z^* direction. The vibrational mode which occurs at 18 Hz is not included in the identification process since this mode of vibration corresponds to the motion of the arm in the $X^* - Y^*$ plane and it is the result of non-central excitation at point 4.

The modal parameters and mode shapes of the arm are obtained by curve fitting the transfer functions using the

¹ A direct transfer function is the ratio of the component of the response in the direction of the excitation and at the point of application of the excitation, to the excitation.

routines of the modal analysis software, "MODAL PLUS", provided by SDRC. The modal parameters and mode shapes of this system obtained from the curve fitted transfer function (Fig. 5.8b) are listed in Tables 5.2 and 5.3, respectively. Notice that four modes of vibration are identified by this curve fitting technique and the natural motions of the arm about its rigid body configuration associated with each mode of vibration are illustrated by Fig. 5.9.

5.2a Identification of Local Stiffnesses

Inspection of the arm's natural motions indicates that the oscillation of the system is produced by the deformations of the shoulder and elbow joints about their axes of rotation and the deflection of the base plate relative to the foundation of the manipulator arm. Therefore, a four degree of freedom model is chosen to represent the T3-776 robot. The relative flexibility of the base plate and foundation can be represented by two concentrated springs, K_1 and K_2 , as shown in Fig. 5.10. The deflections of the shoulder and elbow joints are due to the flexibilities of their corresponding drive assembly which can be described by two springs, K_3 and K_4 , illustrated by Fig. 5.10. Four coordinates are sufficient to fully describe the motion of a four degree

of freedom system. Hence, the coordinates $1Z$ (the motion of point 1 in the Z^* direction), $4Z$, $9Z$, and $11Z$ are selected to represent the motion of the manipulator arm. The modal matrix associated with these coordinates is deduced from Table 5.3 which can be written in the form

$$[\psi] = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 0.399 & -0.44 & 4.51 & 2.34 \\ 0.029 & -0.115 & 0.429 & -2.99 \\ -0.021 & 0.075 & 2.753 & -1.54 \end{bmatrix} . \quad (5.28)$$

Now, we can determine the stiffness matrix associated with $\underline{Z}$ ($1Z$, $4Z$, $9Z$, $11Z$) coordinates using the modal matrix, $[\psi]$, and modal stiffnesses, K_r , given in Table 5.2. The stiffness matrix is obtained from the equation

$$[K^Z] = [\psi]^{-T} [\bar{K}] [\psi]^{-1} . \quad (5.29)$$

It is convenient to transform the stiffness matrix, $[K^Z]$, into a new coordinate system referred to as $\underline{Z\beta}$ coordinates. The $\underline{Z\beta}$ coordinates include, β_3 the rotational motion of the shoulder joint about its axis, β_4 the rotational motion of the elbow joint about its axes, $9Z$, and $11Z$ as shown in Fig. 5.10. This transformation simplifies the identification process since

the stiffness matrix in terms of $\underline{z}_\beta$ coordinates must be a diagonal matrix of the form

$$[K^{z_\beta}] = \begin{bmatrix} K\beta_3 & 0 & 0 & 0 \\ 0 & K\beta_4 & 0 & 0 \\ 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & K_2 \end{bmatrix} \quad . \quad (5.30)$$

Parameter $K\beta_i$ designates the effective stiffness associated with β_i coordinates. It is evident that once $[K^{z_\beta}]$ is obtained, the local stiffnesses are readily identified. The coordinates $\underline{z}$ can be transformed into the coordinates $\underline{z}_\beta$ using the equation

$$\begin{bmatrix} 1z \\ 4z \\ 9z \\ 11z \end{bmatrix} = [TR] \begin{bmatrix} \beta_3 \\ \beta_4 \\ 9z \\ 11z \end{bmatrix} \quad . \quad (5.31)$$

Matrix $[TR]$ is a transformation matrix and its scalar components are functions of the robot geometry. This matrix is defined by the equation

$$[TR] = \left[\begin{array}{cc|cc} [G_1^z]_3 & [G_1^z]_4 & [G_1^z]_1 & [G_1^z]_2 \\ [G_4^z]_3 & [G_4^z]_4 & [G_4^z]_1 & [G_4^z]_2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{cc} [G_9^z]_1 & [G_9^z]_2 \\ [G_{11}^z]_1 & [G_{11}^z]_2 \end{array} \right]^{-1} \quad (5.32)$$

Parameter $[G_j^z]_i$ designates the z component of $[G_j]$ in the i th column. Recalling that the potential energy contribution is independent of the coordinate system, we can write

$$PE = \frac{1}{2} (\underline{Z}_\beta)^T [K^Z_\beta] (\underline{Z}_\beta) = \frac{1}{2} (\underline{Z})^T [K^Z] (\underline{Z}) . \quad (5.33)$$

The substitution of Eqn. (5.31) into (5.33) results in

$$\frac{1}{2} (\underline{Z}_\beta)^T [K^Z_\beta] (\underline{Z}_\beta) = \frac{1}{2} (\underline{Z}_\beta)^T [TR]^T [K^Z] [TR] (\underline{Z}_\beta) . \quad (5.34)$$

By inspection of Eqn. (5.34) we conclude

$$[K^Z_\beta] = [TR]^T [K^Z] [TR] . \quad (5.35)$$

Therefore, the stiffness matrix associated with $\underline{z\beta}$ coordinates can be obtained using the modal parameters deduced from the experimental measurements. The stiffness parameters, K_3 and K_4 , defining the flexibility of the drive assembly are related to the springs, $K_{\beta 3}$ and $K_{\beta 4}$, by the perpendicular distances, R_3 and R_4 , between the joint axes and the axes of corresponding drive assembly

$$K_i = \frac{1}{R_i^2} K_{\beta i} \quad . \quad (5.36)$$

To estimate the values of local stiffnesses more accurately, the identification process is repeated for several different configurations of the arm. The final value of each local stiffness is then obtained by taking the average of the local parameters deduced from the selected configurations of the arm. The identified stiffness parameters of the T3-776 robot are listed in Table 5.4.

5.2b Identification of Local Masses

The mass content of the manipulator arm is represented by three mass elements associated with the base housing, the upper arm, and the lower arm (Fig. 5.11a). The mass of the base housing contains, a mass M_2 concentrated at a point C_2 with coordinates $(15,0,0)$ in terms of the local reference frame, and a mass moment of inertia I_{z2} about the local Z coordinate axis. Parameter M_3 designates the point mass associated with the upper arm which is lumped at the point C_3 $(17.2,0,0)$ in the corresponding local reference frame, and I_{z3} denotes the mass moment of inertia of this link about the local Z axis. The mass content of the lower arm is represented by the point mass M_4 concentrated at the point C_4 $(24,0,0)$ in the associated local coordinate system, and the mass moment of inertia I_{z4} about the local Z axis. Furthermore, the motion of base plate is defined by the psuedo-prosmatic input β_1 and psuedo-revolute input β_2 (Fig. 5.11a). The kinematic representation of this system is shown in Fig. 5.11b and its geometric parameters are listed in Table 5.5. The mass matrix in the Z coordinate system is obtained from the equation

$$[M^Z] = [\psi]^{-T} [\bar{M}] [\psi]^{-1} . \quad (5.37)$$

We can then determine the system inertia matrix, $[I^*]$, by transforming the $\underline{z}$ coordinates into the $\underline{\beta}$ ($\beta_1, \beta_2, \beta_3, \beta_4$) coordinates. This transformation can be defined by the equation

$$\underline{z} = [TR1] \underline{\beta} . \quad (5.38)$$

where

$$[TR1] = \begin{bmatrix} \frac{G_1^z}{G_4^z} \\ \frac{G_9^z}{G_{11}^z} \end{bmatrix} . \quad (5.39)$$

Parameter G_i^z designates the z components of the translational geometric coefficients associated with point i . Recalling that the kinetic energy contribution is independent of the coordinate system, we can write

$$\begin{aligned} KE &= \frac{1}{2} (\dot{\underline{\beta}})^T [I^*] (\dot{\underline{\beta}}) = \frac{1}{2} (\dot{\underline{z}})^T [M^z] (\dot{\underline{z}}) = \\ &\quad \frac{1}{2} (\dot{\underline{\beta}})^T [TR1]^T [M^z] [TR1] (\dot{\underline{\beta}}) . \end{aligned} \quad (5.40)$$

By inspection of Eqn. (5.4) we conclude

$$[I^*] = [TR1] [M^z] [TR1] . \quad (5.41)$$

The system inertia matrix $[I^*]$ is a 4 x 4 symmetric matrix and its scalar components can be obtained from Eqn.

(5.41). To determine the local mass parameters, each independent component of $[I^*]$ must be expressed in terms of the unknown parameters ($M_2, M_3, M_4, I_{z2}, I_{z3}, I_{z4}$).

These expressions can be written in the matrix form

$$[MM] \begin{bmatrix} M_2 \\ M_3 \\ M_4 \\ I_{z2} \\ I_{z3} \\ I_{z4} \end{bmatrix} = \begin{bmatrix} I_{11}^* \\ I_{12}^* \\ I_{13}^* \\ I_{14}^* \\ I_{22}^* \\ I_{23}^* \\ I_{24}^* \\ I_{33}^* \\ I_{34}^* \\ I_{44}^* \end{bmatrix} \quad (5.42)$$

The scalar components of coefficient matrix $[MM]$ are obtained from Eqn. (2.44). Equation (5.42) represents a system of ten equations with six unknowns which can be solved by a least squares method to obtain the local mass parameters of the system. This process can also be performed for several configurations of the arm to provide a more accurate model. The local mass parameters of the T3-776 robot identified by the above procedure are listed in Table 5.4.

5.2c Identification of Local Damping Coefficients

The viscous damping factors, ζ_r , are determined through the curve fitting routine of the measured transfer functions (listed in Table 5.2). Maintaining the basic assumption of proportional damping, we must be able to satisfy the following condition

$$\bar{C}_r = \nu \bar{K}_r . \quad (5.43)$$

Thus, by estimating an approximate value for ν we can determine the damping matrix associated with the modal coordinates. The damping factors, ζ_r , are defined by the equation

$$\zeta_r = \frac{\bar{C}_r}{2(\bar{K}_r \bar{M}_r)^{1/2}} = \frac{\nu \bar{K}_r}{2(\bar{K}_r \bar{M}_r)^{1/2}} = \frac{\nu}{2} \omega_r . \quad (5.44)$$

The final value of ν can be obtained from the average of this coefficient associated with each mode of vibration, or

$$\nu = \left(\sum_{r=1}^4 \frac{2\zeta_r}{\omega_r} \right) / 4 \quad (5.45)$$

Next, the damping matrix $[\bar{C}]$ is evaluated using Eqn. (5.43). Finally, the local damping coefficients are determined by following the procedure of Section 5.2a (these values are listed in Table 5.4).

5.3 Verification of the Identified Parameters

The quality of parametric identification depends on the extent to which the vibrational characteristics of the proposed model match the experimentally determined behavior of the mechanism. The suggested model verification should include, (a) the comparison of the system natural frequencies, deduced from the identified model and experimental data, throughout the arm's workspace, (b) the comparison of the static deflection of the proposed model and the actual system under applied loads, and (c) the comparison of the measured transfer functions with analytically computed transfer functions.

The system natural frequencies are obtained from Eqn. (2.55) using the identified parameters. These frequencies along with the measured resonance frequencies for two geometric configurations of the arm are listed in Tables 5.6 and 5.7. Comparison of the resonance frequencies provides a basis to define the quality of the model based on the overall structural properties of the system.

Static measurements were performed for several configurations in order to determine the total flexibility of the robot arm. A static load was applied to point 1 in the Z^* direction and the deflection of this point was measured by means of a high resolution dial indicator. Furthermore, the total static flexibility of the robot model is determined by computing the joint deflections due to an applied load

$$\underline{\beta} = [K^B]^{-1} [G_1]^T \underline{F}_1 . \quad (5.46)$$

Vector $\underline{F}_1$ designates the magnitude and direction of the load applied to point 1. The total flexibility at point 1 in the Z direction can then be obtained from

$$1Z = [G_1^Z] \underline{\beta} . \quad (5.47)$$

The accuracy of the total static flexibility defines the quality of the identified stiffness parameters regardless of the mass parameters. Table 5.8 presents a list of measured and computed static flexibilities at one of the selected robot configurations.

A comparison of measured and modeled transfer functions provides a basis to verify the identified parameters based on the complete vibration characteristics of the system. The modeled transfer function of the robot

for point 4 in the Z^* direction is obtained by following the procedure introduced in Section 3.3. Figure 5.12 illustrates the measured and modeled transfer functions at a selected robot configuration. Further verification is made by adding external weight to the system and repeating the above procedure (Fig. 5.13).

These comparisons indicate that the proposed model provides a relatively accurate representation for the actual system. The model can be further refined by performing more accurate measurements and considering other significant flexible components such as bending flexibility of the link elements.

5.4 Instrumentation

In recent years, the advent of high performance, low cost minicomputers, and computing techniques such as the fast Fourier transform have given birth to powerful new "instruments" known as digital Fourier analyzers. The ability of these machines to quickly and accurately provide the frequency spectrum of a time-domain signal has opened a new era in structural dynamics testing. It is now relatively simple to obtain fast, accurate, and complete measurements of the dynamic behavior of mechanical structures, via transfer function measurements and modal analysis.

Techniques have been developed which now allow the modes of vibration of an elastic structure to be identified from measured transfer function data. Once a set of transfer (frequency response) functions relating points of interest on the structure have been measured and stored, they may be operated on to obtain

i.e., the natural frequency, damping factor, and characteristic mode shape for the predominant modes of vibration of the structure. Most importantly, the modal responses of many modes can be measured simultaneously and complex mode shapes can be directly identified. [31]

The instrumentation package selected to conduct the test was the GEN RAD 2515 computer aided test system. This system was selected because it would meet the requirements of high baseband resolution, bias protection, interactive data-acquisition, large CPU memory, and sophisticated curve-fitting software compatibility. A summary of the specifications for the test equipments are presented here.

Modal Analyzer

Genrad 2515-9715 Computer Aided Test System

Eight input A/D channels with programmable anti-biasing filters

LSI 11/23 controller with RT-11 operating system

512 Kilobytes of CPU memory

Mini-Winchester rigid disk drive with 10 megabyte total capacity

Mini-Floppy disk (5-1/4) with .5 megabyte total

Sensitivity: 8 ranges (62.5 mV, .125 V, .25 V, .5 V, 1 V, 2 V, 4 V, 8 V)

Bandwidths: Selectable in powers of 2 times
1.25 Hz to 20 KHZ, plus 25 KHZ

Frequency Lines: 320 alias-free lines for every
1024 input points in frame

Frame Size: Selectable in powers of 2 from 526 to
8192 points/channel

SDRC Modal-Plus software

Impactor

PCB model 086B20 short sledge hammer, S.No. 625

Sensitivity: 1.04 mV/lb

Range: 0-500 lb

PCB model 084A60 Impact super soft tip

Accelerometer

PCB model 393C seismic accelerometer, S.No. 412

Sensitivity: 1.007 V/g

PCB model 480 D06 power supply, S.No. 4088

5.5 Improvement of System Design

It is possible to improve the structural integrity of the T3-776 robot by increasing its fundamental natural frequency as discussed in Section 2.7. The simplest way to improve the resonance frequency of this system is to increase the modal stiffness associated with the first

vibrational mode. The system's fundamental frequency is defined by Rayleigh's quotient

$$\omega_1^2 = \frac{K_1}{M_1} \quad . \quad (5.48)$$

In order to increase the value of ω_1 to a desired value ω_1^d , the modal stiffness K_1 must be increased to K_1^d , or

$$(\omega_1^d)^2 = \frac{K_1^d}{M_1} \quad . \quad (5.49)$$

Suppose, it is desired to increase the resonance frequency by ten percent. We can write

$$(1.1 \omega_1)^2 = \frac{K_1^d}{M_1} \quad . \quad (5.50)$$

Substituting Eqn. (5.48) into (5.50) results in

$$K_1^d = 1.21 K_1 \quad . \quad (5.51)$$

The stiffness matrix associated with the β coordinates is defined by the equation

$$[K^{\beta}] = \begin{bmatrix} 1745079 & -2380463 & 0 & 0 \\ & 5.8300865E8 & 0 & 0 \\ & & 335431 \times R_3^2 & 0 \\ & \text{Symmetric} & & 471433 \times R_4^2 \end{bmatrix}. \quad (5.52)$$

The eigenvector corresponding to the first mode of vibration is found to be

$$\underline{\psi}^{(1)} = \begin{bmatrix} 1.0 \\ -0.33 \\ -2.7 \\ -1.9 \end{bmatrix}. \quad (5.53)$$

The modal stiffness $\bar{K}_1^d$ is defined by the summation

$$\bar{K}_1^d = \sum_{i=1}^4 \sum_{j=1}^4 [K^{\beta}]_{i;j} \psi_i^{(1)} \psi_j^{(1)}. \quad (5.54)$$

Therefore, the desired modal stiffness can be obtained by increasing the scalar components of $[K^{\beta}]$. By inspecting the coefficients of $[K^{\beta}]_{i;j}$ in Eqn. (5.54), it is clear that the stiffness parameters associated with the shoulder and elbow joints are the prominent factors in increasing the modal stiffness. We can then select these two parameters to improve the system design. Furthermore, the

effective stiffness at the shoulder and elbow joints are geometrically dependent parameters. Thus, the simplest way to increase these parameters is to improve the effective geometric parameters. This can be achieved by increasing the magnitude of R_3 and R_4 . Rearranging Eqn. (5.54) results in

$$(335431) (2.7)^2 R_3^2 + (471433) (1.9)^2 R_4^2 =$$

$$\bar{K}_1 d - \sum_{i=1}^2 \sum_{j=1}^2 [K^B]_{i,j} \psi_i^{(1)} \psi_j^{(1)}. \quad (5.55)$$

Parameters R_3 and R_4 can be determined by choosing an appropriate value for either R_3 , or R_4 and computing R_4 or R_3 using Eqn. (5.55). In this case, R_4 , at a selected configuration, is chosen to be 7.572 (an increase of 1.0 in. from the previous value) and R_3 is computed to be 12.53 (an increase of 1.16 in.). Using these parameters, the eigenvalues of the improved system are computed and compared with the original system design (Tables 5.9 and 5.10). It is possible to further improve the system design by decreasing the flexibility of drive assemblies or improving the mass distribution of the arm.

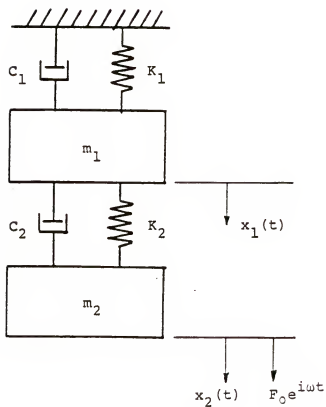


Figure 5.1. The Two Degree-of-Freedom System.

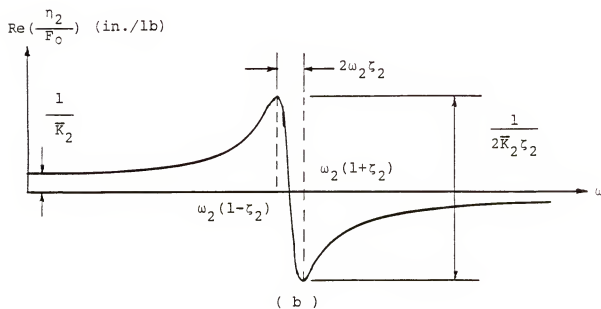
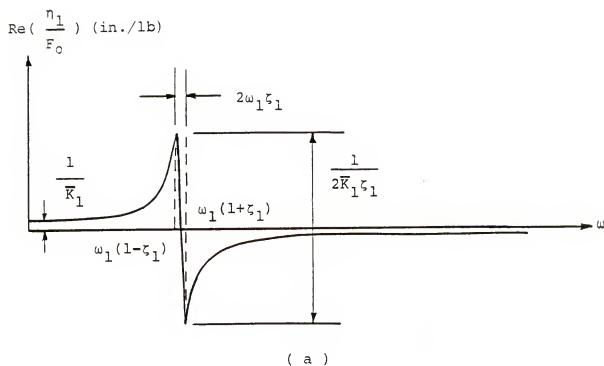


Figure 5.2. System Transfer Functions in the Modal Coordinates.

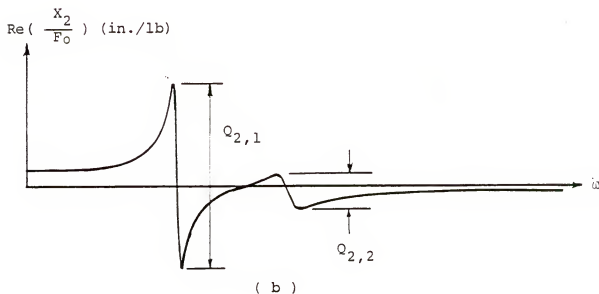
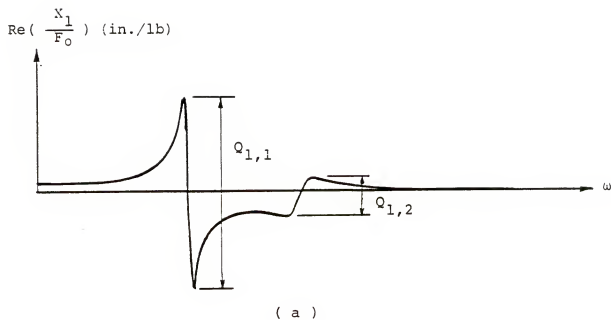


Figure 5.3. System Transfer Functions in the Generalized Coordinates.

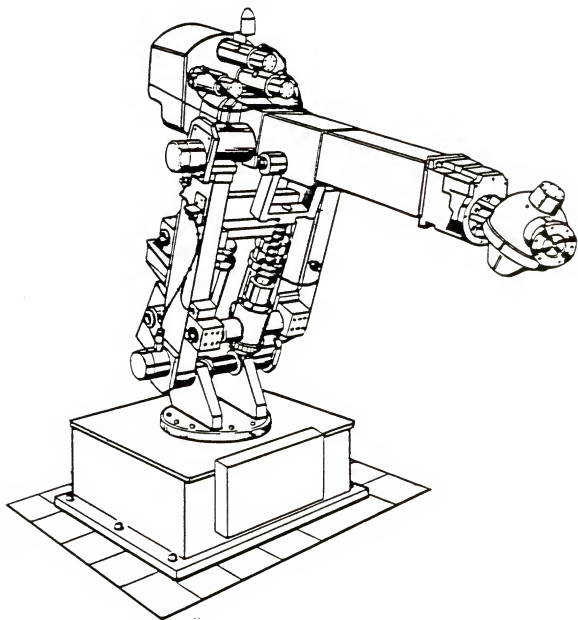


Figure 5.4. Cincinnati Millicron T3-776 Robot Manipulator.

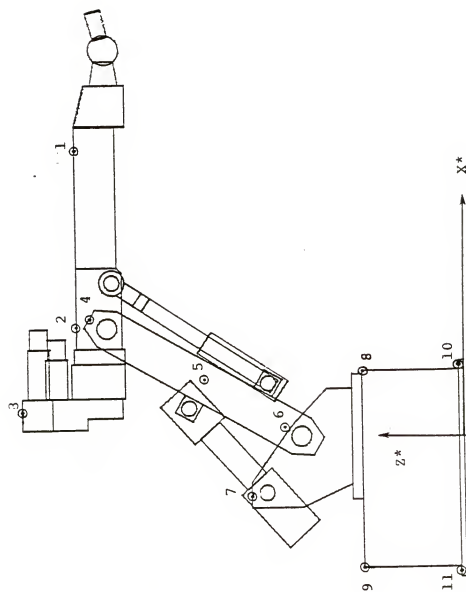


Figure 5.5. Measured Point Locations (scale 1/18).

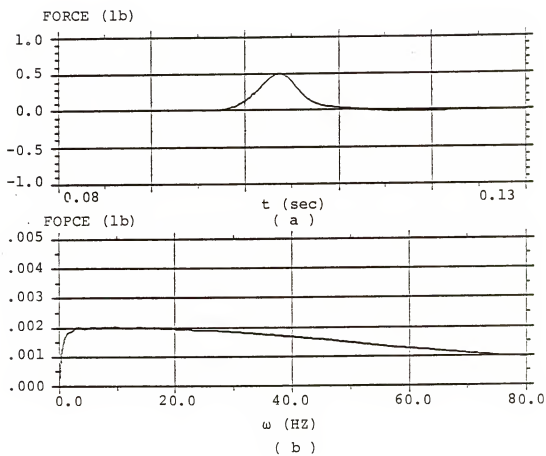


Figure 5.6. Display of Impulse and its Fourier Transform.

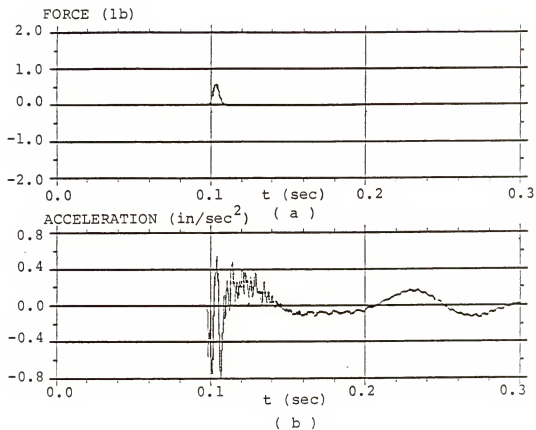


Figure 5.7. Manipulator's Response to an Impulsive Exciting Force.

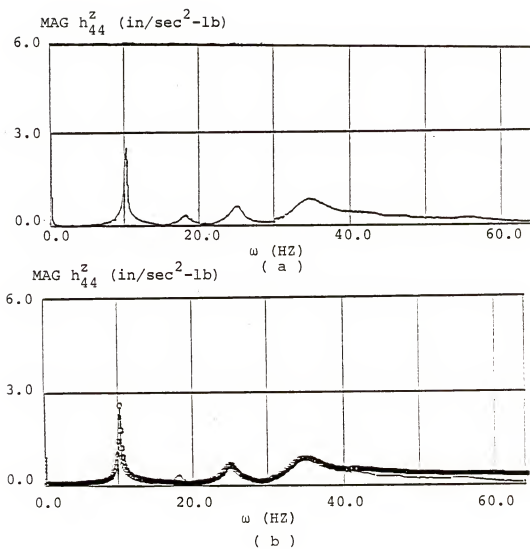
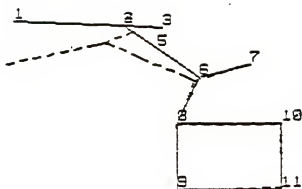
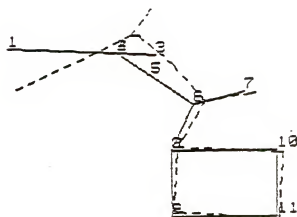


Figure 5.8. Curve Fitting of System Transfer Function.



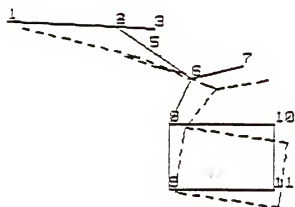
10.09 Hz

(a)



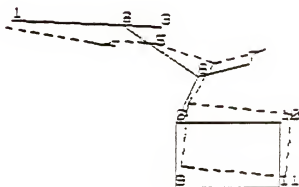
24.83 Hz

(b)



34.24 Hz

(c)



40.64 Hz

(d)

Figure 5.9. The Manipulator's Mode Shapes.

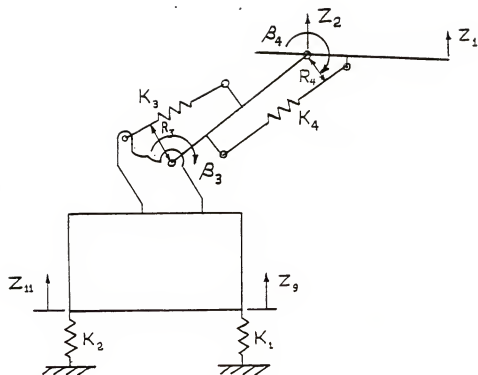


Figure 5.10. The Representative Four Degree-of-Freedom Model.

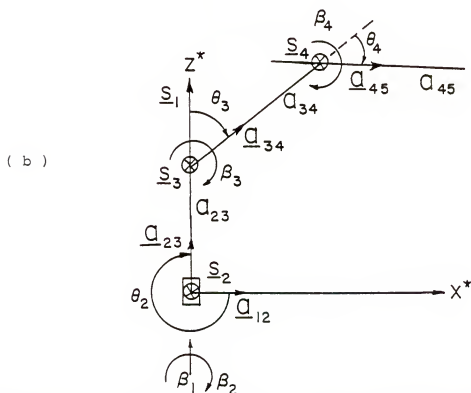
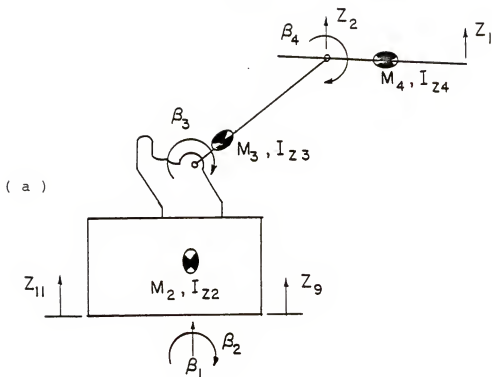
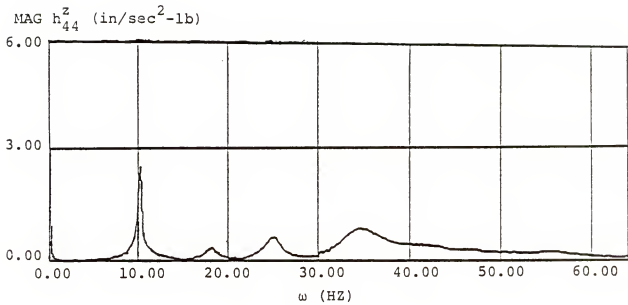
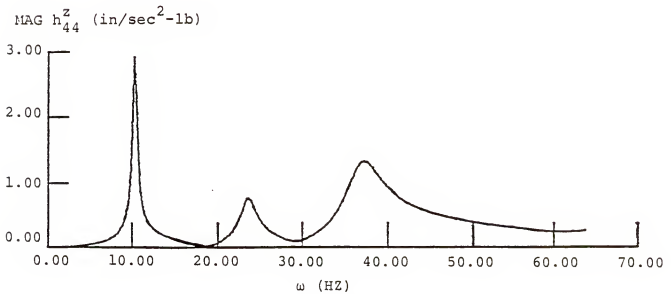


Figure 5.11. Planar Representation of the T3-776 Robot.

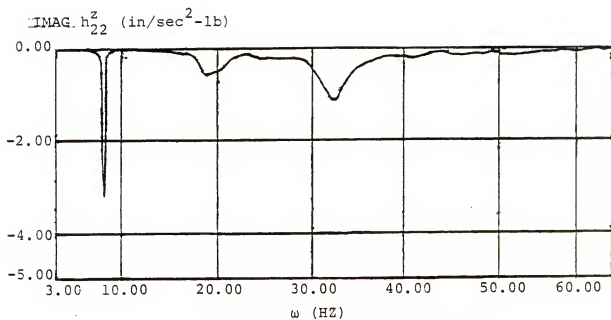


(a) Measured Transfer Function

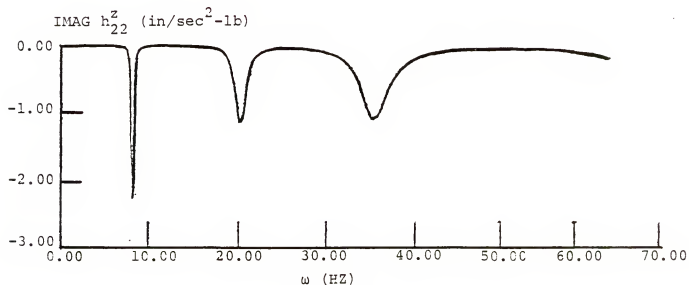


(b) Modeled Transfer Function

Figure 5.12. Direct Transfer Function at 4Z.



(a) Measured Transfer Function



(b) Modeled Transfer Function

Figure 5.13. Transfer Function at 2Z with Additional Mass.

Table 5.1. Coordinates of the Measured Points.

Point	X*(in.)	Y*(in.)	Z*(in.)
1	74.80	0.00	64.25
2	34.85	0.00	62.40
3	20.85	0.00	61.75
4	34.85	0.00	62.40
5	22.70	0.00	53.60
6	4.85	0.00	40.75
7	-11.00	0.00	47.25
8	15.00	0.00	25.00
9	15.00	0.00	0.00
10	-23.00	0.00	25.00
11	-23.00	0.00	0.00

Table 5.2. Modal Parameters.

ω_r (HZ)	$\bar{K}_r$ (lb/in)	$\bar{M}_r$ (lb-sec ² /in)	$\bar{\zeta}_r$
10.09	5.435×10^3	1.35	0.0138
24.83	7.497×10^4	3.08	0.031
34.24	5.896×10^6	127.38	0.06
40.64	2.005×10^7	307.45	0.055

Table 5.3. Mode Shapes of the Example Manipulator

ω_r	Point	ΔX^*	ΔY^*	ΔZ^*
10.09 Hz	1	-4.381E-1	0.00	1.000E0
	2	-4.281E-1	0.00	3.900E-1
	3	-6.265E-1	0.00	9.259E-2
	4	-3.381E-1	0.00	3.995E-1
	5	-3.247E-1	0.00	2.384E-1
	6	-9.732E-2	0.00	7.083E-2
	7	-8.231E-2	0.00	-1.074E-2
	8	-3.000E-2	0.00	2.484E-2
	9	-4.975E-3	0.00	2.869E-2
	10	-2.918E-2	0.00	-1.563E-2
	11	-6.714E-3	0.00	-2.142E-2
24.83 Hz	1	1.963E-1	0.00	1.000E0
	2	3.438E-1	0.00	-4.315E-1
	3	-1.202E-1	0.00	-1.091E0
	4	5.040E-1	0.00	-4.400E-1
	5	5.131E-1	0.00	-2.390E-1
	6	2.759E-1	0.00	-1.010E-1
	7	2.611E-1	0.00	1.874E-2
	8	1.410E-1	0.00	-9.297E-2
	9	4.344E-2	0.00	-1.155E-1
	10	1.219E-1	0.00	6.804E-2
	11	3.705E-2	0.00	7.469E-2

Table 5.3. Continued

ω_r	Point	ΔX^*	ΔY^*	ΔZ^*
34.24 Hz	1	2.213E0	0.00	1.000E0
	2	2.629E0	0.00	4.398E0
	3	3.552E0	0.00	6.732E0
	4	1.347E0	0.00	4.509E0
	5	1.992E0	0.00	2.876E0
	6	3.494E0	0.00	1.556E0
	7	3.679E0	0.00	2.032E0
	8	2.083E0	0.00	5.229E-1
	9	7.773E-1	0.00	4.294E-1
	10	1.796E0	0.00	2.725E0
	11	6.361E-1	0.00	2.753E0
40.65 Hz	1	-8.585E-1	0.00	1.000E0
	2	-2.972E0	0.00	2.748E0
	3	-6.299E0	0.00	2.690E0
	4	-1.304E0	0.00	2.339E0
	5	-8.651E-2	0.00	-6.515E-1
	6	2.048E0	0.00	-1.944E0
	7	2.275E0	0.00	-2.153E0
	8	1.602E0	0.00	-2.711E0
	9	8.239E-1	0.00	-2.990E0
	10	1.444E0	0.00	-1.283E0
	11	7.649E-1	0.00	-1.539E0

Table 5.4. Identified Parameters

i	K_i (lb/in)	M_i (lb-sec ² /in)	I_{zi} (lb-sec ² -in)
1	335431	0	0
2	471433	8.04	911.72
3	1118876	9.68	171.31
4	626203	0.78	1003.87

Table 5.5. Geometrical Parameters

j	s_{jj} (in.)	a_{jk} (in.)	α_{jk} (deg.)	θ_j (deg.)
1	0	0	270	0
2	0	37.25	0	270
3	0	44	0	54.89
4	0	40	—	33.34

Table 5.6. Comparison of Predicted and Measured Natural Frequencies for Configuration A

ω	Predicted	Measured	Error
1	10.28	10.09	1.8%
2	23.52	24.83	5.3%
3	36.85	34.24	7.6%
4	74.46	-	-

Table 5.7. Comparison of Predicted and Measured Natural Frequencies for Configuration B

ω	Predicted	Measured	Error
1	11.67	11.22	3.9%
2	26.42	28.30	6.6%
3	44.08	39.67	11.1%
4	78.40	-	-

Table 5.8. Comparison of Predicted and Measured Static Flexibilities for Configuration A

Predicted (in/lb)	Measured (in/lb)	Error
2.07×10^{-4}	2.06×10^{-4}	0.48%

Table 5.9. Comparison of Original and Improved Natural Frequencies for Configuration A

ω	Original	Improved	Improvement
1	10.28	11.23	9.2%
2	23.52	25.76	9.5%
3	36.85	38.22	3.7%
4	74.46	76.26	2.4%

Table 5.10. Comparison of Original and Improved Natural Frequencies for Configuration B

ω	Original	Improved	Improvement
1	11.67	12.43	6.5%
2	26.42	28.44	7.6%
3	44.08	45.05	2.2%
4	78.40	81.46	3.9%

CHAPTER VI

ILLUSTRATIVE EXAMPLES

This chapter details the application of the mathematical model to a common industrial robot by computer simulation. Additionally, the system deformation due to inertia loads is determined using both the quasi-static formulation and the complete dynamic equations of motion. The comparison of the system deformation deduced from the two methods at several operating speeds illustrates the effect of the manipulator's operational frequency on the system deflection.

6.1 Description of Example Manipulator

The Cincinnati Milacron manipulator represents a common type of manipulator in industrial use today. A drawing of its physical form and its six controllable rotational freedoms is shown in Fig. 6.1. It is an RR-R-RRR manipulator, where the R denotes a revolute joint and R-R-R indicates that these three successive joint axes are parallel, and therefore allows only planar motion. The deformational freedoms of the arm are represented by twelve pseudo-inputs, β_1 through β_{12} , as shown in Fig. 6.2. The

revolute pseudo-inputs, β_1 , β_2 , β_6 , β_{10} , β_{11} , and β_{12} , describe the deformational motion of the manipulator's joints about their axes of rotation. The motion of the base housing relative to the foundation and flexibility of the housing components are assumed to be negligible. The bending deformation of the upper arm in the local X-Z plane is described by the prismatic psuedo-input, β_3 , and revolute pseudo-input β_4 . The torsional deformation of this component is represented by β_5 . The prismatic and revolute pseudo-inputs, β_7 and β_8 , describe the bending deformation of the lower arm in the corresponding local X-Z plane. Parameter, β_9 , represents the torsional deformation of the lower arm. The bending deformation of the upper and lower arms in their corresponding X-Y plane and the axial deformation of these components are neglected. Furthermore, the link components associated with the manipulator's wrist are assumed to be rigid. Figure 6.2 illustrates the kinematic representation of the example manipulator and Table 6.1 lists the kinematic parameters for this arm.

The inertia content of the example manipulator is described by the mass, rotational inertia, and location of the mass for each link. Table 6.2 presents the list of these inertia properties. The stiffness content of the manipulator arm is described by the stiffness matrix for each joint-link combination. These stiffness matrices are listed in Table 6.3.

6.2 System Natural Frequencies and Transfer Functions

In this section, the natural frequencies and the corresponding mode shapes of the arm at a selected configuration are determined. The system inertia matrix is obtained from Eqn. (2.44) and the stiffness matrix is constructed using the submatrices associated with each joint-link combination as described by Eqn. (2.57). The resonance frequencies and eigenvectors are then computed using the QR method. Tables 6.4 and 6.5 list the system eigenvalues and eigenvectors, respectively.

The first six mode shapes of the manipulator arm are shown in Fig. 6.3. The 5.6 HZ mode is the motion of the arm in the X^*-Z^* plane produced by the in phase vibration of the shoulder and elbow joints. The 9.5 HZ mode is the motion of the arm in the X^*-Y^* plane caused mainly by the oscillation of the first, third, and seventh psuedo-inputs. The 22 HZ mode is a higher mode to the 5.6 HZ with the elbow joint vibrating out of phase to the motion of the shoulder joint. The 24 HZ mode is the motion of the arm in the X^*-Y^* plane and is the result of the flexure and torsional vibration of the upper and lower arms. The 34 HZ mode is a higher mode to the 9.5 HZ, the 37 HZ mode is the vibration of the arm in the X^*-Z^* plane dominated by the oscillation of the elbow and wrist joints.

In order to determine the transfer functions for this

manipulator, the system is excited by a harmonic force, $\underline{f}_H$, applied to a point H located on the arm's end effector. The magnitude and direction of the exciting force are defined by the equation

$$\underline{f}_H = \underline{F}_H e^{i\omega t} = (1,1,1)^T e^{i\omega t} \quad (6.1)$$

The damping matrix is assumed to be proportional to the stiffness matrix by a factor of .0005. Using the procedure described in Section 3.3, the direct transfer function associated with point H is determined. The components of this transfer function in terms of the global coordinate system are shown in Fig. 6.4 (real parts) and Fig. 6.5 (imaginary parts). This transfer function indicates that the magnitude of system vibration associated with the sixth mode and above is less than two percent of the system vibration associated with the first five modes. Therefore, only five modes of vibration contribute to the system oscillation and the other vibrational modes are insignificant.

6.3 Manipulator's Dynamic Response

As mentioned previously, any discontinuity in a time derivative of the motion of the manipulator arm will excite one or more vibrational modes of the system. In order to

investigate the reaction of the example manipulator to a rapid change in the velocity of the end-effector, an impulsive force, $\underline{F}_H \delta(t)$, is applied to the point H located on the manipulator's terminal device. The exciting force is described by the equation

$$\underline{F}_H \delta(t) = (1, 1, 1)^T \delta(t) \quad (6.2)$$

The motion of point H, produced by this exciting force can then be obtained by following the procedure discussed in Section 3.4.

To examine the effect of each mode of vibration on the system response, the dynamic response is computed by considering the superposition of several consecutive normal modes. Figure 6.6a represents the X component of the response produced by the first four modes of vibration. The dynamic response in the X direction produced by the first six modes of vibration is shown in Fig. 6.6b. A comparison of these results with the response of the system produced by all twelve modes of vibration (Fig. 6.6c) indicates that the dynamic response can be adequately described by at most six modes of vibration. The components of the response in Y and Z directions are shown in Figs. 6.7 and 6.8, respectively.

In order to determine the system response produced by a rapid change in the acceleration of the arm's end-effector, the exciting force, $\underline{F}_H$, is represented by a step function,

where

$$\underline{f}_H = \underline{F}_H^u(t) = (1,1,1)^T u(t) \quad (6.3)$$

Figures 6.9 through 6.11 respectively illustrate the dynamic response of point H in the X^* , Y^* , and Z^* directions, produced by the first two, first four, and all twelve modes of vibration. By comparing these results, we can conclude that only the first two modes of vibration are sufficient to define the response of this system due to a step function exciting force.

The system response due to the impulsive exciting force, $\underline{F}_H^{\delta}(t)$, is determined using the nonlinear differential equations of motion defined by Eqn. (3.77). A comparison of this response and the one deduced from the linear equations of motion illustrates the effect of Coriolis and centripetal accelerations on the manipulator's dynamic response (Figs. 6.12 through 6.14). As seen in these Figures, the effect of the nonlinear terms on the system response is insignificant.

In conclusion, the vibration behavior of a manipulator arm can adequately be described by a linear superposition of the first few modes of vibration. Therefore, it is sufficient to determine the first few system eigenvalues and eigenvectors to investigate the behavior of the arm due to an exciting phenomena.

6.4 Deformation Due to the Gross Motion

The inertia loads resulting from the nominal motion of a manipulator arm cause the arm to deform about its rigid body position. The magnitude and behavior of the deformational motion depend on the operational speed of the arm. A manipulator's operational speed is described by a parameter, ω , which indicates the rate of moving a load from one position to another and back. This section presents an investigation of the deformation of the example manipulator due to its gross motion at several operational speeds.

Assume that the manipulator task is to transfer an object from one work station to another along a straight line in a plane parallel to the X^*-Y^* plane. The initial position of the hand is described by a point $P_1(70,0,59)$ and the final position is specified by $P_2(77,7,59)$. The path length function relative to time is defined by the 3-4-5 polynomial shown in Fig. 6.15.

Three operational speeds are considered for obtaining the arm's deformation. In the first case, the representative frequency of operation, ω , is chosen to be 20 percent of the fundamental frequency. The displacement of manipulator's end-effector from its specified path function is computed using the quasi-static analysis. This deformation is then compared with the resultant deformation obtained from the complete dynamic equations of motion (Eqn.

(4.11)). Figures 6.18 through 6.20 respectively show the hand deformations in the X^* , Y^* , and Z^* directions obtained from the two methods.

In the second case, the operational frequency is assumed to be 10 percent of the manipulator's fundamental frequency (Fig. 6.16). The hand deformations for this case are shown in Figs. 6.21 through 6.23. Finally, the hand deformations produced by the gross motion of the arm at a frequency of $.022\omega_1$ (Fig. 6.17) are presented in Figs. 6.24 through 6.26.

At the operational frequency of $0.2\omega_1$, the resultant deformations obtained from the quasi-static analysis are off from the actual deformations by an average of seventy percent (Table 6.6). This error will reduce to forty percent as the operational frequency is decreased to $0.1\omega_1$ (Table 6.7). Table 6.8 shows that the error at the operational frequency of $0.022\omega_1$ is only eight percent. Thus, the comparison of the resultant deformations obtained from the quasi-static method and the complete dynamic equations of motion indicates that as the operating speed increases with respect to the fundamental frequency, the manipulator will oscillate about its static equilibrium position. However, the system deflections can be predicted by the quasi-static method, if the mechanism operating speed is less than five percent of the system fundamental frequency.

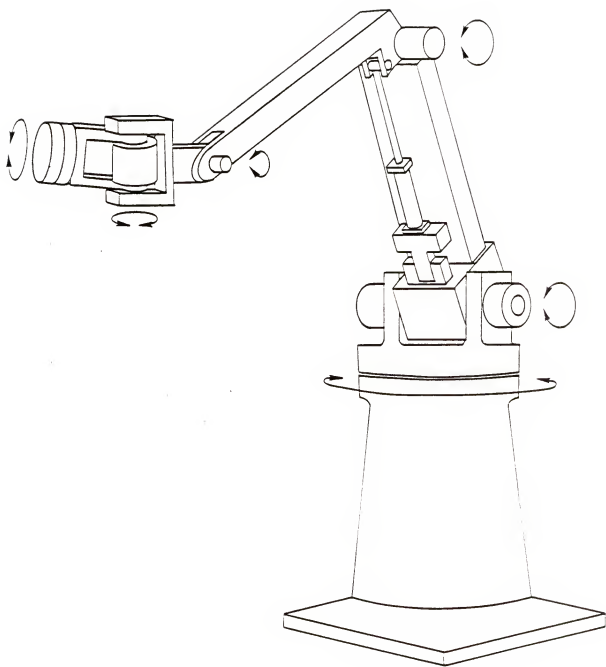


Figure 6.1. Representative 6R Manipulator.

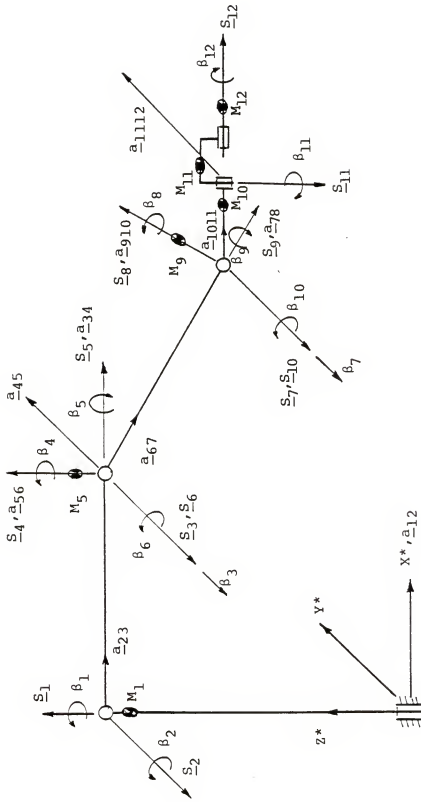


Figure 6.2. Kinematic Representation of the Example Manipulator.

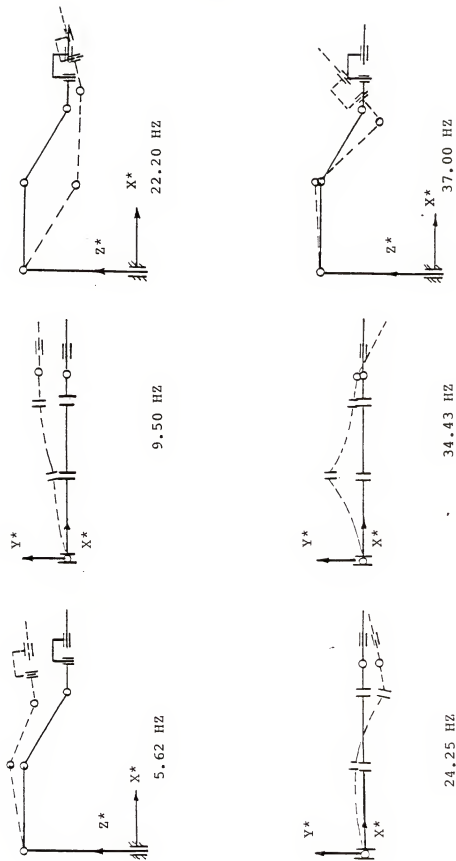
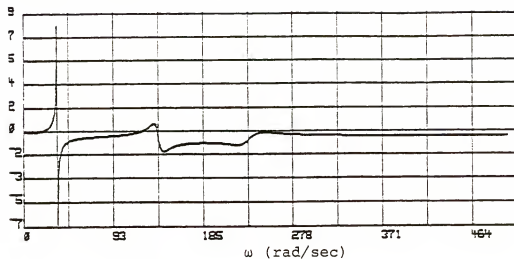


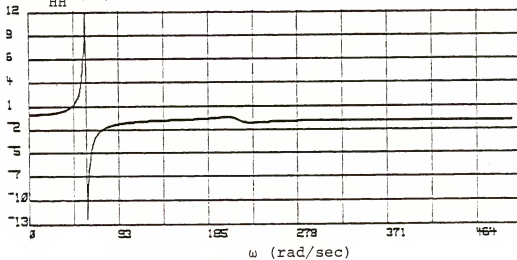
Figure 6.3. Display of the Mode Shapes.

Re h_{HH}^x (in/sec²-lb)

157



Re h_{HH}^y (in/sec²-lb)



Re h_{HH}^z (in/sec²-lb)

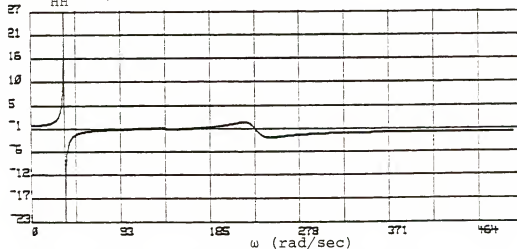


Figure 6.4. Direct Transfer Function at H.

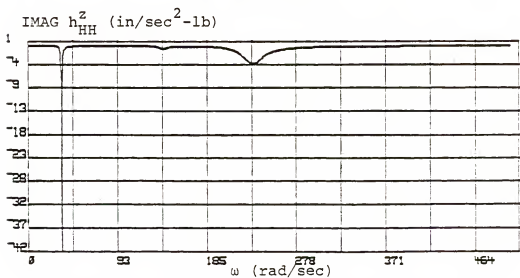
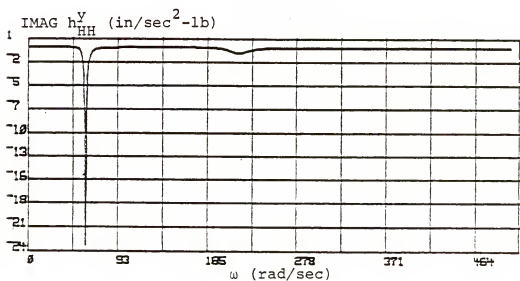
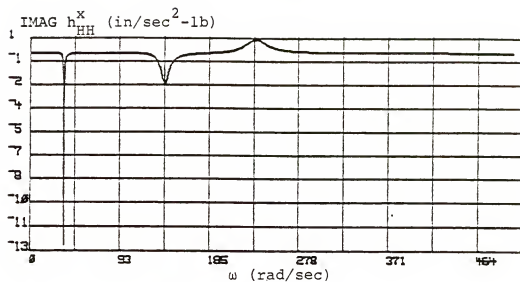
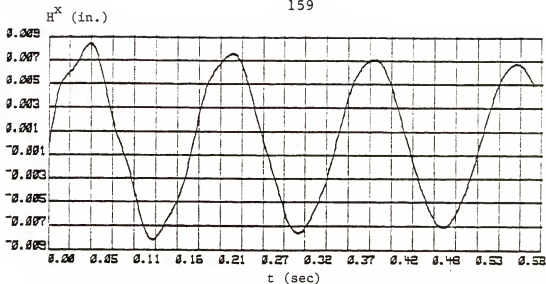
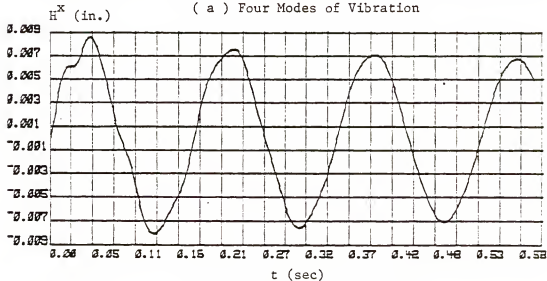


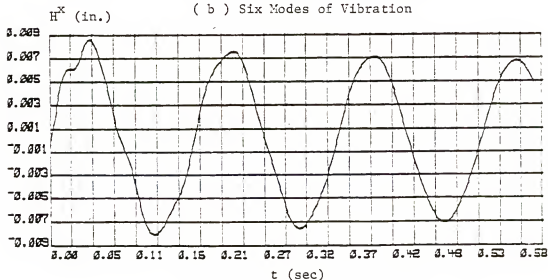
Figure 6.5. Direct Transfer Function at H.



(a) Four Modes of Vibration

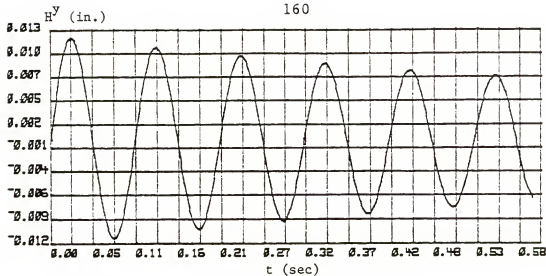


(b) Six Modes of Vibration

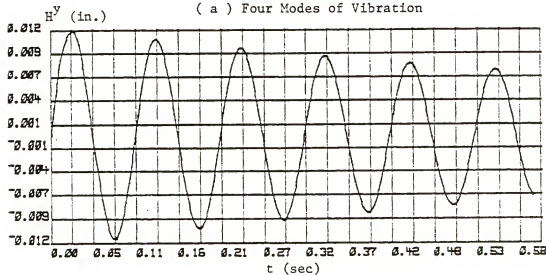


(c) Twelve Modes of Vibration

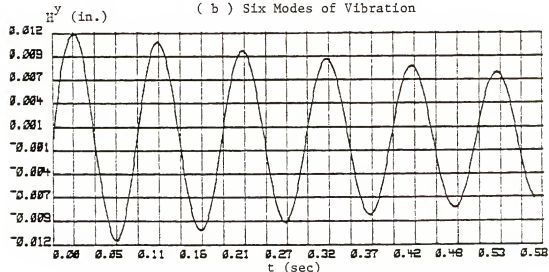
Figure 6.6. Manipulator's Response to a Unit Impulse.



(a) Four Modes of Vibration

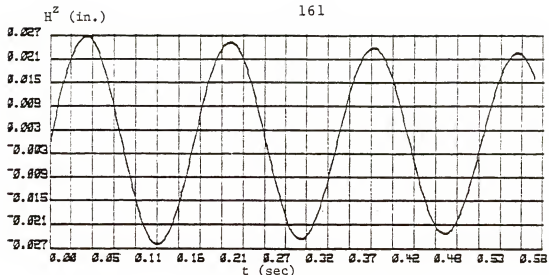


(b) Six Modes of Vibration

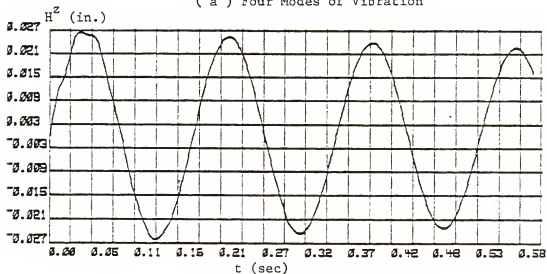


(c) Twelve Modes of Vibration

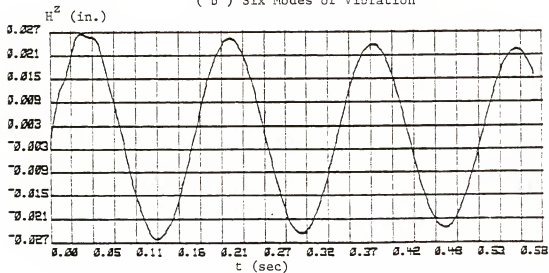
Figure 6.7. Manipulator's Response to a Unit Impulse.



(a) Four Modes of Vibration

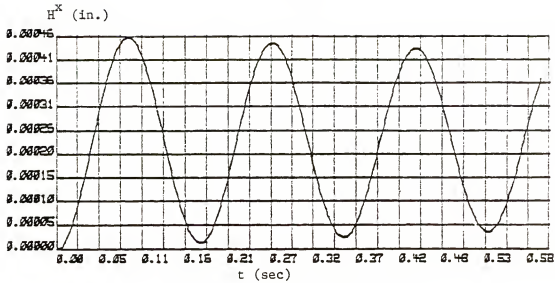


(b) Six Modes of Vibration

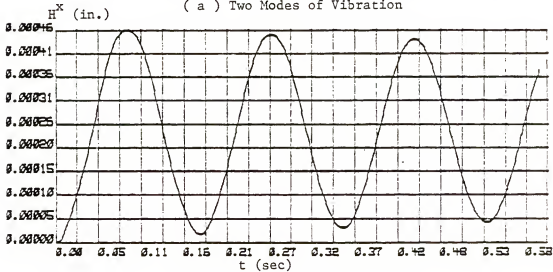


(c) Twelve Modes of Vibration

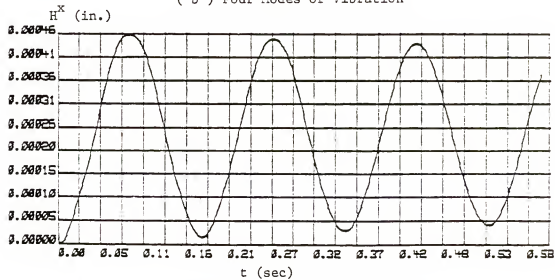
Figure 6.8. Manipulator's Response to a Unit Impulse.



(a) Two Modes of Vibration

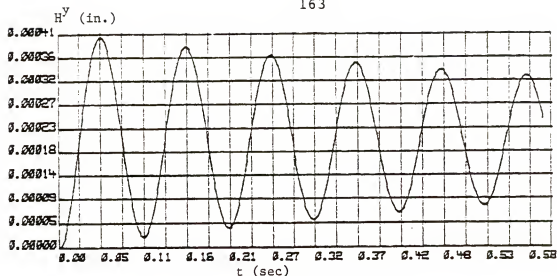


(b) Four Modes of Vibration

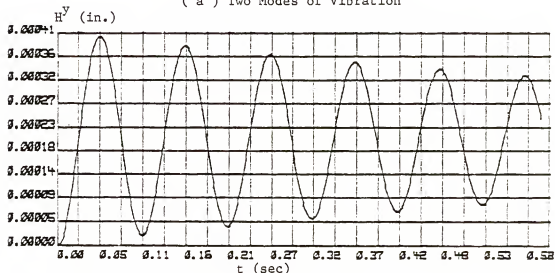


(c) Twelve Modes of Vibration

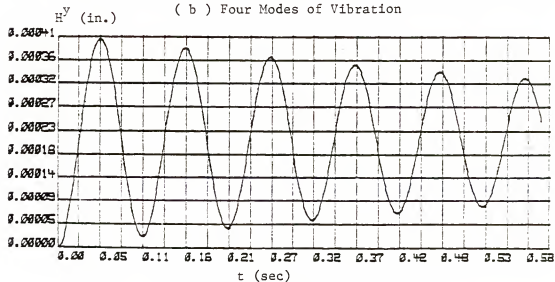
Figure 6.9. Manipulator's Response to a unit step.



(a) Two Modes of Vibration

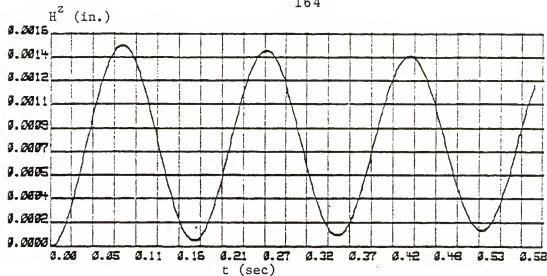


(b) Four Modes of Vibration

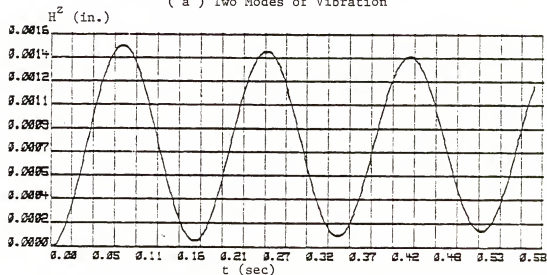


(c) Twelve Modes of Vibration

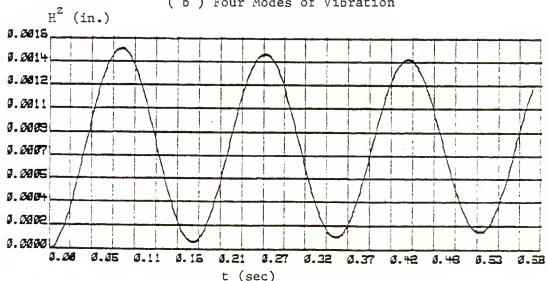
Figure 6.10. Manipulator's Response to a Unit Step.



(a) Two Modes of Vibration

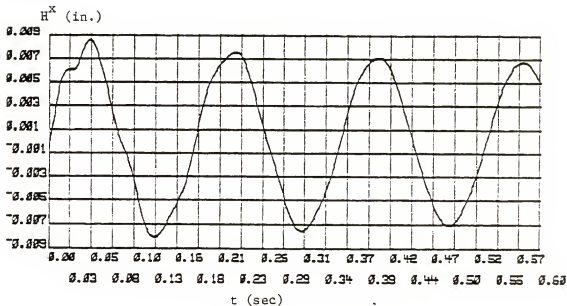


(b) Four Modes of Vibration

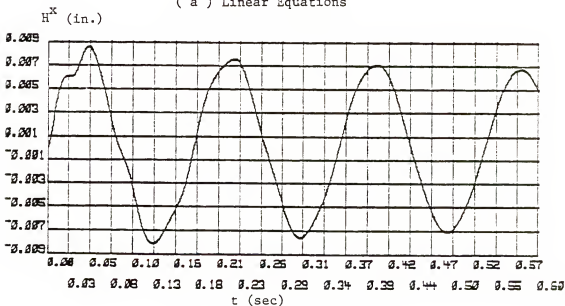


(c) Twelve Modes of Vibration

Figure 6.11. Manipulator's Response to a Unit Step.

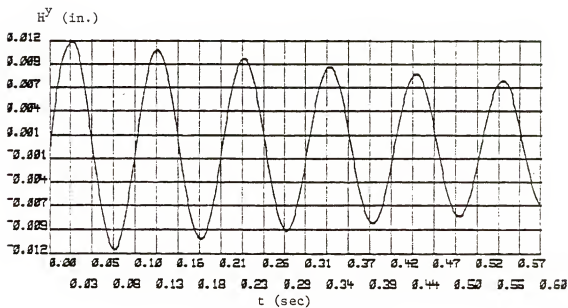


(a) Linear Equations

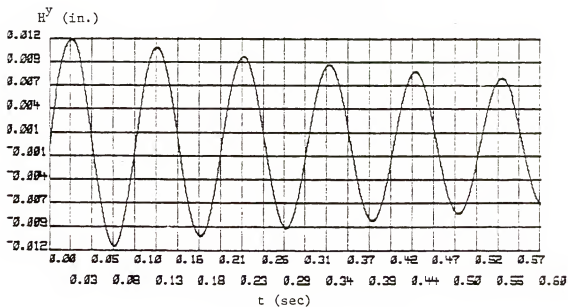


(b) Nonlinear Equations

Figure 6.12. Manipulator's Response to a Unit Impulse.

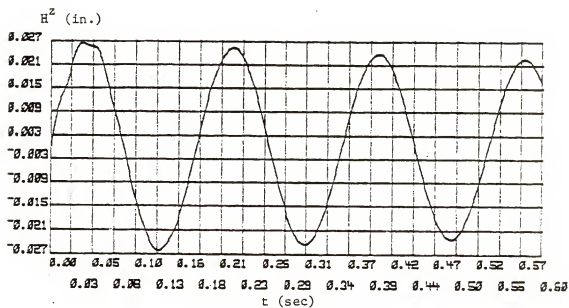


(a) Linear Equations

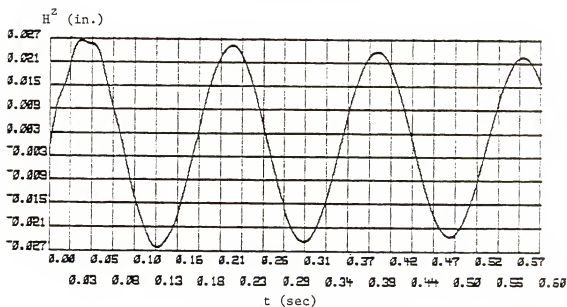


(b) Nonlinear Equations

Figure 6.13. Manipulator's Response to a Unit Impulse.



(a) Linear Equations



(b) Nonlinear Equations

Figure 6.14. Manipulator's Response to a Unit Impulse.

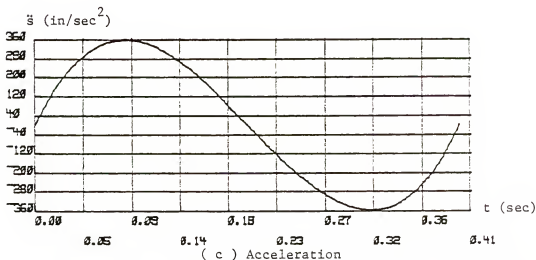
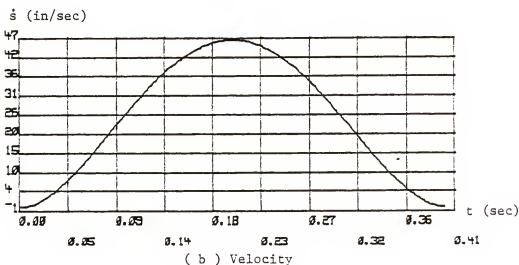
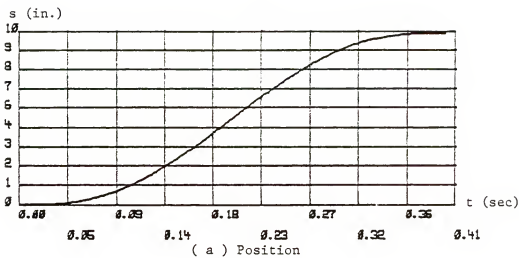


Figure 6.15. The Path Function for Operational Speed of $0.2\omega_1$.

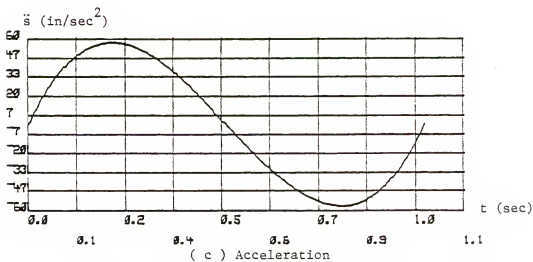
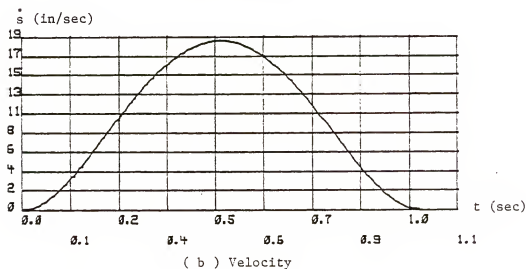
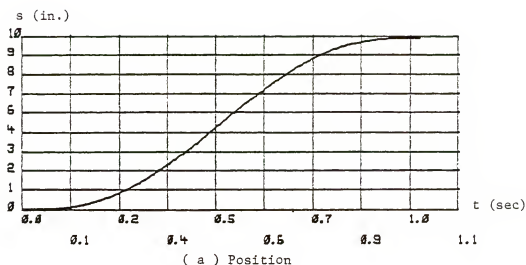
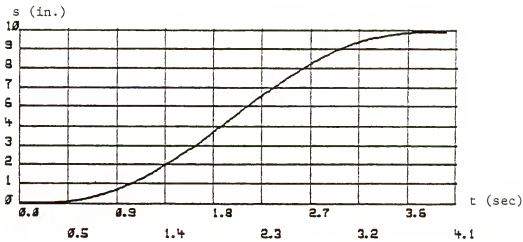
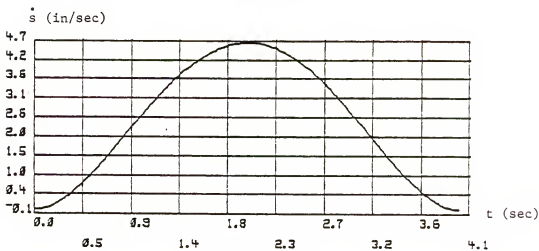


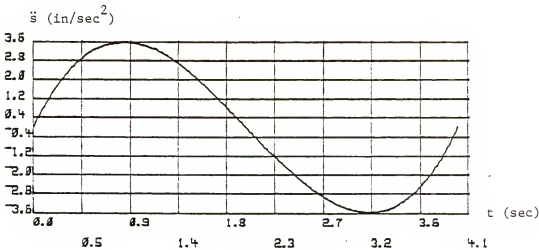
Figure 6.16. The Path Function for Operational Speed of $0.1\omega_1$.



(a) Position

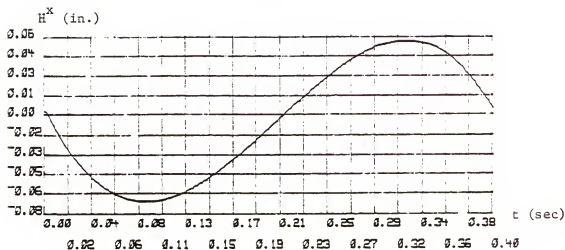


(b) Velocity

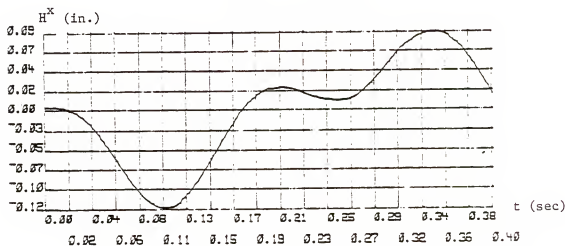


(c) Acceleration

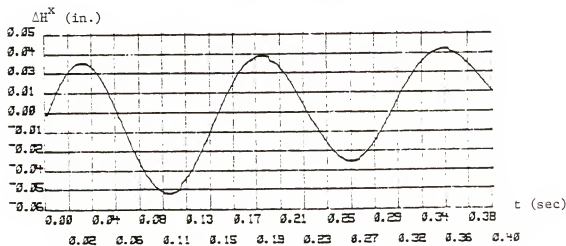
Figure 6.17. The Path Function for Operational Speed of $0.022\omega_1$.



(a) Quasi-Static Analysis



(b) Dynamic Analysis



(c) The Difference Between (a) and (b)

Figure 6.18. Hand Deflection for Operational Speed of $0.2\omega_1$.

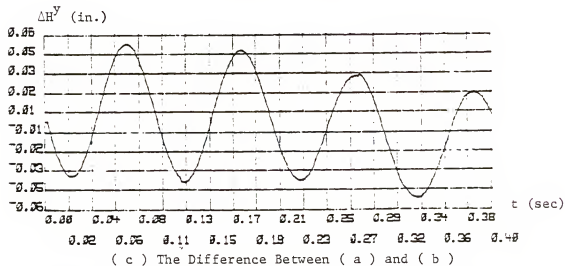
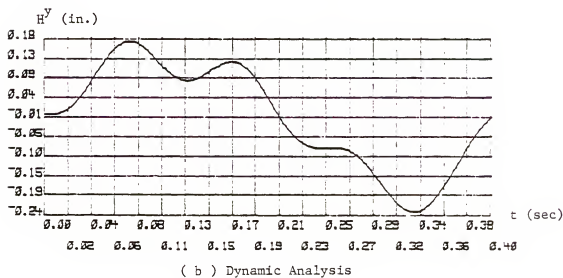
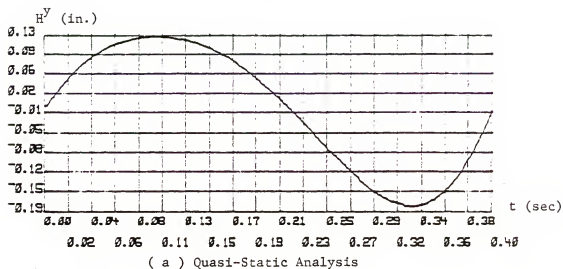


Figure 6.19. Hand Deflection for Operational Speed of $0.1\omega_1$.

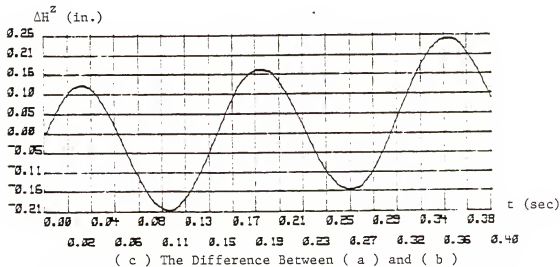
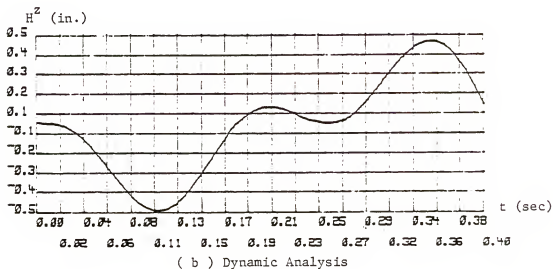
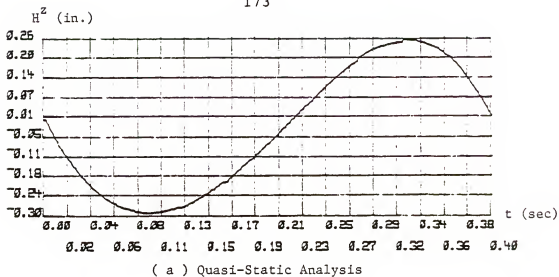
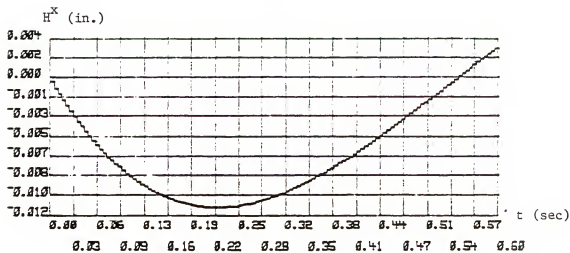
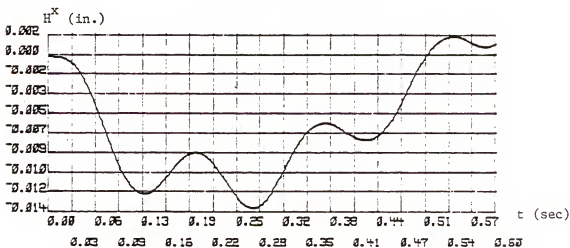


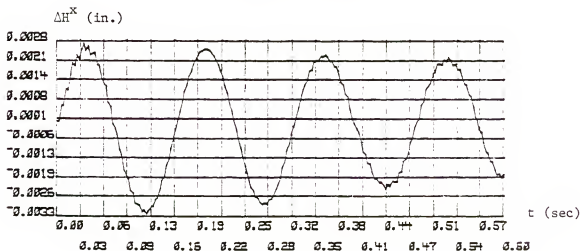
Figure 6.20. Hand Deflection for Operational Speed of $0.2\omega_1$.



(a) Quasi-Static Analysis

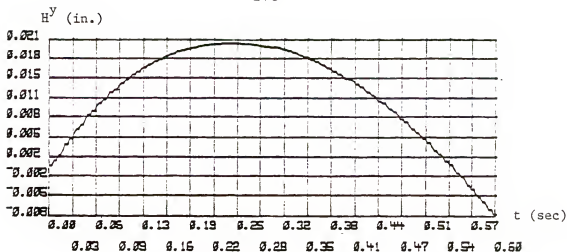


(b) Dynamic Analysis

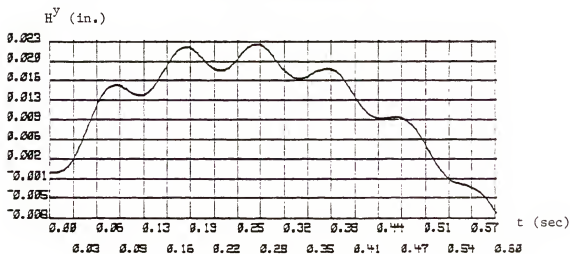


(c) The Difference Between (a) and (b)

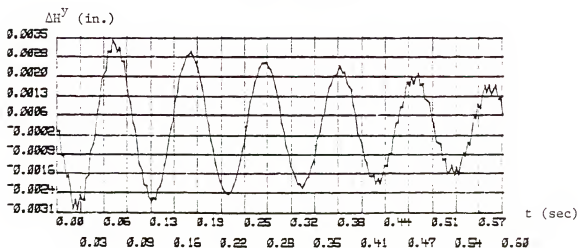
Figure 6.21. Hand Deflection for Operational Speed of $0.1\omega_1$.



(a) Quasi-Static Analysis

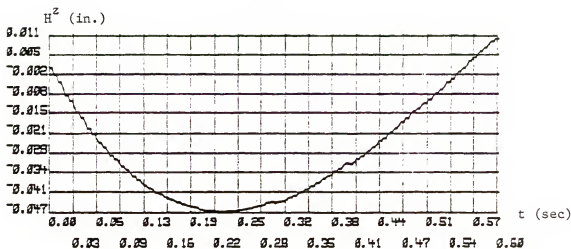


(b) Dynamic Analysis

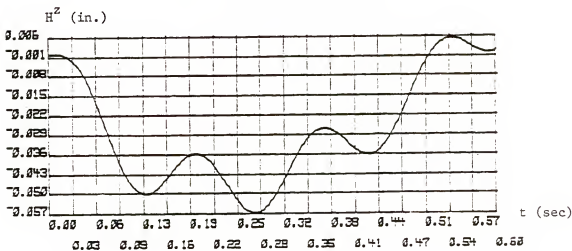


(c) The Difference Between (a) and (b)

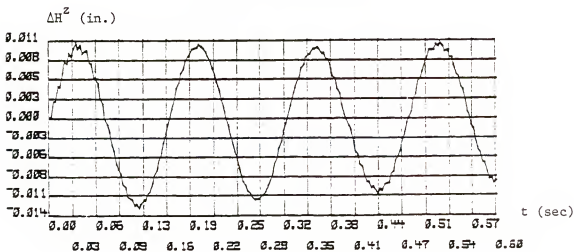
Figure 6.22. Hand Deflection for Operational Speed of $0.1\omega_1$.



(a) Quasi-Static Analysis

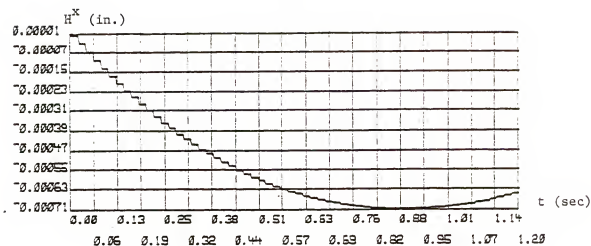


(b) Dynamic Analysis

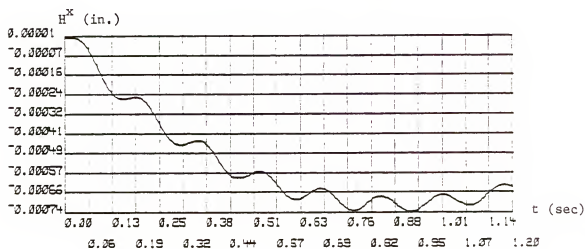


(c) The Difference Between (a) and (b)

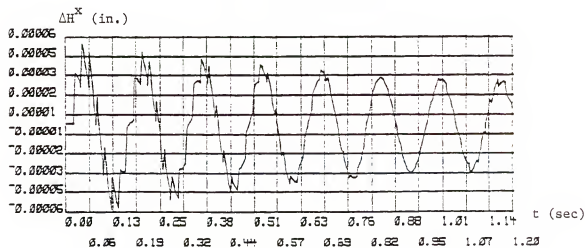
Figure 6.23. Hand Deflection for Operational Speed of $0.1\omega_1$.



(a) Quasi-Static Analysis



(b) Dynamic Analysis



(c) The Difference Between (a) and (b)

Figure 6.24. Hand Deflection for Operational Speed of $0.022\omega_1$.

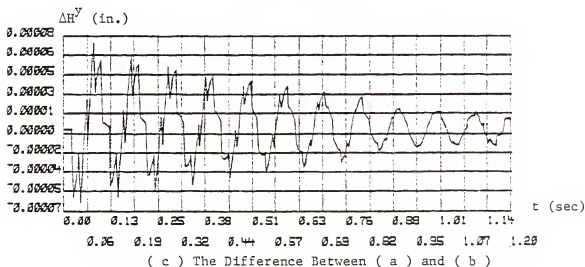
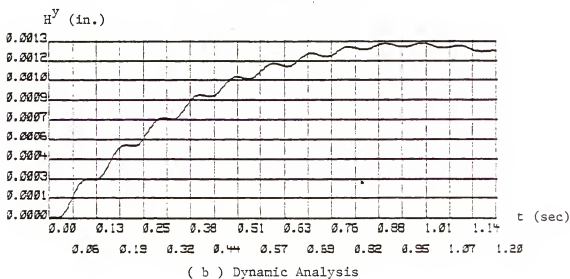
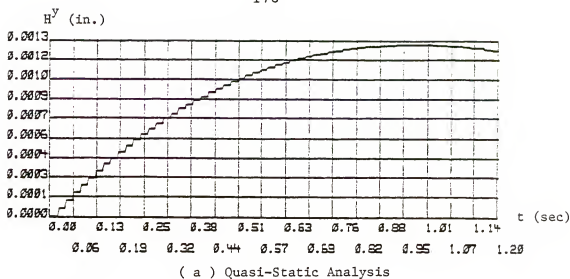
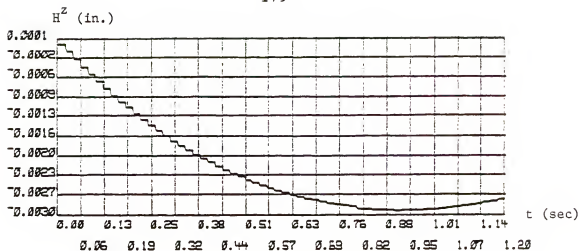
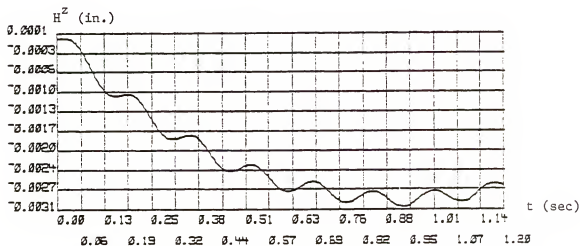


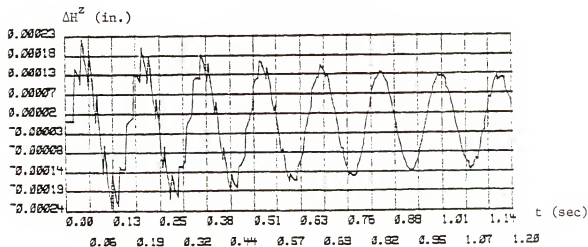
Figure 6.25. Hand Deflection for Operational Speed of $0.022\omega_1$.



(a) Quasi-Static Analysis



(b) Dynamic Analysis



(c) The Difference Between (a) and (b)

Figure 6.26. Hand Deflection for Operational Speed of $0.022\omega_1$.

Table 6.1. Kinematic Parameters of the Example Manipulator

j	s_{jj} (in.)	a_{jk} (in.)	α_{jk} (deg.)	θ_j (deg.)
1	59	0	90	0
2	0	40	0	0
3	0	0	270	0
4	0	0	90	90
5	0	0	270	90
6	0	40	0	240
7	0	0	270	0
8	0	0	90	90
9	0	0	270	90
10	0	8	90	300
11	0	0	90	90
12	13	0	—	0

Table 6.2. Inertia Properties for the Example Manipulator

j	x(j)	Centroid y(j) (in.)	z(j)	M _j (lb-sec ² /in)	I _{xj}	I _{yj}	I _{zj}
1	0	0	-9	5.7	0	0	800
5	0	0	0	0.67	300	700	700
9	0	0	0	0.62	300	700	700
10	3	0	0	0.31	20	40	40
11	0	0	-2	0.21	20	40	40
12	0	0	3	0.13	20	40	40

Table 6.3. Stiffness Matrices

$$[K_{12}] = [40 \times 10^6]$$

$$[K_{45}] = [5 \times 10^6]$$

$$[K_{56}] = [5 \times 10^6]$$

$$[K_{67}] = [5 \times 10^6]$$

$$[K_{23}] = \begin{bmatrix} 20 \times 10^6 & 0 & 0 & 0 \\ & 0.55 \times 10^6 & -10.85 \times 10^6 & 0 \\ & \text{Symmetric} & 2.89 \times 10^8 & 0 \\ & & & 48 \times 10^6 \end{bmatrix}$$

$$[K_{34}] = \begin{bmatrix} 20 \times 10^6 & 0 & 0 & 0 \\ & 0.55 \times 10^6 & -10.85 \times 10^6 & 0 \\ & \text{Symmetric} & 2.89 \times 10^8 & 0 \\ & & & 48 \times 10^6 \end{bmatrix}$$

Table 6.4. System Eigenvalues

r	ω_r (HZ)	r	ω_r (HZ)
1	5.62	7	45.22
2	9.50	8	54.97
3	22.19	9	64.80
4	24.15	10	72.99
5	34.42	11	257.44
6	37.05	12	485.50

Table 6.5. System Eigenvectors

ψ (1)	ψ (2)	ψ (3)	ψ (4)	ψ (5)	ψ (6)	ψ (7)	ψ (8)	ψ (9)	ψ (10)	ψ (11)	ψ (12)
0	1	0	1	1	0	1	1	1	1	1	1
1	0	1	0	0	1	0	0	0	0	0	0
0	1.35	0	31.05	-80.47	0	-86.19	2.01	-7.03	410	48.72	92.43
0	0.11	0	1.44	-3.65	0	-3.95	0.01	-0.45	18.47	-0.63	-6.04
0	0.16	0	-8.08	-2.28	0	-1.45	-1.96	-0.47	-21.74	0.33	-0.99
1.03	0	-1.9	0	0	-11.77	0	0	0	0	0	0
0	-2.05	0	8.18	-52.62	0	10.85	66.73	94.8	-266	15.4	-262
0	-0.07	0	0.27	-2.59	0	0.34	3.02	4.23	-11	-0.91	6.12
0	-0.04	0	-5.3	0.9	0	1.63	1.82	1.42	42.5	-0.12	-1.62
0.17	0	-0.41	0	0	120.2	0	0	0	0	0	0
0	0.1	0	-3.1	16.96	0	-15.53	5.51	5.51	-15.67	-0.4	1.2
0	0	0	-2.83	-1.68	0	0.23	23.34	-11.68	-23.6	0.24	-0.67

Table 6.6. Hand deformation for operational speed
of $0.2\omega_1$

t (sec)	dynamic analysis	quasi-static	percent error
	$ \Delta H \times 10^{-5} \text{ (in.)}$	$ \Delta H \times 10^{-5} \text{ (in.)}$	
.000	.000	.000	.000
.025	3133.517	16055.681	-412.385
.050	18892.943	26148.998	-38.406
.075	39582.652	31456.987	20.528
.100	51405.873	32480.618	36.815
.125	46868.065	30230.267	35.499
.150	28083.078	24828.010	11.591
.175	12489.363	17295.326	-38.480
.200	9778.729	8250.398	15.629
.225	9201.661	2543.651	72.357
.250	8218.406	12522.078	-52.366
.275	9366.332	21920.976	-134.040
.300	23233.425	28778.706	-23.868
.325	43292.683	31745.485	26.672
.350	51218.246	28580.997	44.198
.375	39471.156	18430.902	53.305
.400	10709.820	1719.814	33.942

Table 6.7. Hand deformation for operational speed
of $0.1\omega_1$

t (sec)	dynamic analysis	quasi-static	percent error
	$ \Delta H \times 10^{-5} (\text{in.})$	$ \Delta H \times 10^{-5} (\text{in.})$	
.000	.000	.000	.000
.040	710.825	1624.717	-128.568
.080	3422.217	3054.532	10.744
.120	5288.867	4082.298	22.813
.160	4887.031	4744.860	2.909
.200	4214.801	5093.020	-20.837
.240	5258.330	5216.652	.793
.280	6254.241	5053.317	19.202
.320	4690.716	4779.259	-1.888
.360	3338.311	4222.004	-26.471
.400	3581.543	3513.464	1.901
.440	3695.945	2767.252	25.127
.480	1960.470	1818.481	7.243
.520	325.449	849.641	-161.067
.560	439.289	288.423	34.343
.600	730.696	1309.819	-79.256

Table 6.8. Hand deformation for operational speed
of $0.022\omega_1$

t (sec)	dynamic analysis	quasi-static	percent error
	$ \Delta H \times 10^{-5} (\text{in.})$	$ \Delta H \times 10^{-5} (\text{in.})$	
.000	.000	.000	.000
.080	49.642	46.325	6.683
.160	114.233	101.545	11.107
.240	147.853	149.613	-1.191
.320	201.276	190.908	5.151
.400	225.750	225.819	-.030
.480	265.164	255.144	3.779
.560	275.438	278.478	-1.104
.640	302.985	296.554	2.122
.720	306.459	309.722	-1.065
.800	325.191	318.314	2.115
.880	323.377	325.678	-.711
.960	328.918	326.041	.875
1.040	318.727	322.672	-1.238
1.120	318.482	315.832	.832
1.200	301.956	305.768	-1.263

CHAPTER VII

CONCLUSION

It has been demonstrated that a lumped-parameter model can provide a reliable representation of flexible manipulator arms. This model drastically simplifies the dynamic analysis of these complex mechanisms. A generic formulation has been presented using the geometric influence coefficients to describe the vibrational behavior of the system. The deformational freedoms of the arm are represented by a set of imaginary inputs referred to as psuedo-inputs. The geometric coefficients for these imaginary inputs are then generated. This allows the formulation of the equations of motion describing oscillatory motion of the system. This formulation is simple to implement on a digital computer. The designer need only consider the dominant, flexible components which allows him to select a simple, yet sufficient representation of the mechanism.

The lowest natural frequencies of vibration are a good indicator of the overall structural integrity of the manipulator system and determine the maximum cycle speed for performing precise arm motion. Therefore, the fundamental

frequency is utilized as a design criteria to increase the system rigidity, leading to precision at higher operational speeds. This frequency is described explicitly in terms of local parameters to demonstrate the effect of each parameter on the magnitude of fundamental frequency. System flexibility can then be decreased by improving the most significant local parameters. However, drastic changes of local parameters may cause a change in the overall vibration behavior of the system; thus the improvement of the local parameters must be limited in order not to alter the vibrational behavior of the mechanism.

The small motion response of manipulator arms can be described by a set of linear, differential equations by neglecting the Coriolis and centripital accelerations. The investigation of the effect upon the system response of these nonlinear terms relative to the linear terms, indicated that the nonlinear terms comprise less than five percent of the total small motion response of these mechanisms. Furthermore, it was demonstrated that the system response can be adequately described by the superposition of the first few modes of vibration. Although a few modes of vibration can be utilized to determine the small motion response, all significant deformational freedoms must be considered for developing the equations of motion.

For precision control of a manipulator, it is necessary to predict and compensate for the system deformation due to

the inertia loads produced by the nominal motion of the arm. As shown in Chapter IV the nominal motion of the arm has no significant effect on the manipulator's natural frequencies. Thus it is sufficient to determine the resonance frequencies in a series of stationary configurations. The system deflection can be predicted by a quasi-static method if the mechanism operating cycle speed is less than five percent of the system fundamental frequency. However, the complete dynamic equations of motion must be used for higher operational speeds. A numerical technique was attempted for solving the complete dynamic equations of motion. This was unsatisfactory, since it required long computational times. In order to overcome this difficulty a more practical technique should be developed for solving these equations. It may be convenient to assume that the dynamic equations represent a vibratory system with nonproportional damping and then apply the modal analysis method to solve these equations. Further investigation is required to demonstrate the effect of various path functions (i.e. constant acceleration curve, trapezoidal curve, etc.) on the system deformation.

Experimental modal analysis is a reliable, and effective method for identifying the local, dynamic parameters of a manipulator arm. The application of this method has been demonstrated by defining the local parameters of an industrial manipulator arm. A four degree

of freedom model was chosen to represent the planar motion of T3-776 industrial robot. The comparison of the vibrational behavior of the representative model and actual system showed that this model can be utilized to describe the dynamic characteristics of T3-776 robot. Although the final results were quite satisfactory, there was a great deal of difficulty associated with numerical evaluation of the local parameters. It is suspected that the main reason for this difficulty is the inadequacy of the proposed model.

A model which includes the bending flexibilities of the upper and lower arms may prove to be an adequate representation for this manipulator arm. Furthermore, the vibration measurements were performed using a uni-directional accelerometer which makes it impossible to measure the motion of a specific point in three directions. Therefore, to avoid errors associated with the description of the motion of a specific point in space, a tri-directional accelerometer should be used.

APPENDIX A

ARM ACCELERATION AND SECOND-ORDER INFLUENCE COEFFICIENTS

The angular acceleration of a body in space is defined by the time derivative of its angular velocity. For serial manipulators, the angular acceleration of link j can then be obtained from

$$\underline{a}_{jk} = \frac{d}{dt} \underline{\omega}_{jk} = \left(\frac{d}{dt} [G_{jk}] \right) \dot{\underline{\phi}} + [G_{jk}] \ddot{\underline{\phi}} \quad (A.1)$$

Differentiating the n th component of $[G_{jk}]$ results in

$$\frac{d}{dt} [G_{jk}]_n = \begin{cases} \dot{\underline{s}}_n & , \quad n \leq j; \quad n\text{-revolute} \\ 0 & , \quad \text{otherwise} \end{cases} \quad (A.2)$$

where

$$\dot{\underline{s}}_n = \underline{\omega}_{(n-1)n} \times \underline{s}_n = \sum_{m=1}^{n-1} \dot{\theta}_m \underline{s}_m \times \underline{s}_n \quad (A.3)$$

Equation (A.2) can be rewritten in a matrix form by introducing the second-order rotational influence coefficient $[H_{jk}]$

$$\frac{d}{dt} [G_{jk}]_n = \dot{\underline{\phi}}^T [H_{jk}]_{;n} \quad (A.4)$$

where, $[H_{jk}]$ is an $N \times N$ array and each component of this array is a vector of length three. The components of $[H_{jk}]$ are defined by the equation

$$[H_{jk}]_{m;n} = \frac{\partial}{\partial \dot{\phi}_m} \left(\frac{d}{dt} [G_{jk}]_n \right) = \frac{\partial}{\partial \dot{\phi}_m} [G_{jk}]_n . \quad (A.5)$$

Substituting Eqn. (A.4) into (A.1), we can write

$$\underline{a}_{jk} = \dot{\underline{\phi}}^T [H_{jk}] \dot{\underline{\phi}} + [G_{jk}] \ddot{\underline{\phi}} . \quad (A.6)$$

From Eqns. (A.2), (A.3), and (A.5), the components of the second-order rotational influence coefficients are evaluated as

$$[H_{jk}]_{m;n} = \begin{cases} \underline{S}_m \times \underline{S}_n & , \quad m < n \leq j; \quad m,n\text{-revolute} \\ 0 & , \quad \text{otherwise} . \end{cases} \quad (A.7)$$

The components of the second-order influence coefficients, $[\hat{H}_{jk}]$, for a flexible manipulator which is deflected from its rigid body position are simply defined by the equation

$$[\hat{H}_{jk}]_{m;n} = \begin{cases} \hat{\underline{S}}_m \times \hat{\underline{S}}_n & , \quad m < n \leq j; \quad m,n\text{-revolute} \\ 0 & , \quad \text{otherwise} . \end{cases} \quad (A.8)$$

Analogously, the transitional acceleration of a point, P, can be defined by the equation

$$\underline{\ddot{a}}_P = \underline{\dot{\phi}}^T [H_P] \underline{\dot{\phi}} + [G_P] \underline{\ddot{\phi}} . \quad (A.9)$$

The components of the second-order translational influence coefficients, $[H_P]$, are defined by the equation

$$[H_P]_{m;n} = \frac{\partial}{\partial \phi_m} \left(\frac{d}{dt} [G_P]_n \right) = \frac{\partial}{\partial \phi_m} [G_P]_n . \quad (A.10)$$

These terms can be readily evaluated by differentiating the expressions in Eqn. (2.28) with respect to time and rewriting the time derivatives in terms of the joint velocity. Table A.1 represents the expressions for the second-order influence coefficients for rigid manipulators which depend on the joint types and sequence. These expressions for flexible manipulators, which are deflected from their rigid body position, are shown in Table A.2.

Table A.1. Second-Order Influence Coefficients for Serial Manipulators

Symbol	Joint type:m	Joint type:n	Restrictions	Value
Rotational Influence Coefficients				
$[H_{jk}]_{m;n}$	R	R	$m < n \leq j$	$S \times S_{\overline{m} \overline{n}}$
	R	R	$m > n$ or $n > j$	0
	P	P	all m, n	0
	R	P	all m, n	0
Translational Influence Coefficients				
$[H_j]_{p \ m;n}$	R	R	$m < n \leq j$	$S \times \{S \times (P - R_{\overline{n}})\}$
	R	R	$n < m \leq j$	$S_{\overline{n}} \times \{S \times (P - R_{\overline{n}})\}$
	P	R	$m < n \leq j$	0
	P	R	$n < m \leq j$	$S_{\overline{n}} \times S_{\overline{m}}$
	R	P	$m < n \leq j$	$S \times S_{\overline{m} \overline{n}}$
	R	P	$n < m \leq j$	0
	P	P	all m, n	0
	-	-	$(m \text{ or } n) > j$	0

Table A.2. Second-Order Influence Coefficients for Flexible Serial Manipulators

Symbol	Joint type:m	Joint type:n	Restrictions	Value
Rotational Influence Coefficients				
$[H_{jk}]_{m,n}$	R	R	$m < n \leq j$	$\hat{S}_{-m} x \hat{S}_{-n}$
	R	R	$m > n$ or $n > j$	0
	P	P	all m, n	0
	R	P	all m, n	0
Translational Influence Coefficients				
$[H_p]_{m,n}$	R	R	$m < n \leq j$	$\hat{S}_{-m} x \{ \hat{S}_{-n} x (\hat{p} - \hat{R}_{-n}) \}$
	R	R	$n < m \leq j$	$\hat{S}_{-n} x \{ \hat{S}_{-m} x (\hat{p} - \hat{R}_{-m}) \}$
	P	R	$m < n \leq j$	0
	P	R	$n < m \leq j$	$\hat{S}_{-n} x \hat{S}_{-m}$
	R	P	$m < n \leq j$	$\hat{S}_{-m} x \hat{S}_{-n}$
	R	P	$n < m \leq j$	0
	P	P	all m, n	0
	-	-	$(m \text{ or } n) > j$	0

APPENDIX B
EXPRESSIONS FOR INERTIA
POWER MATRIX

This appendix presents the calculation of the inertia power matrix, derived by Thomas [29]. This matrix is defined by the equation

$$[P_n] = \left(\frac{\partial}{\partial \underline{\phi}} [I^*]_n; \right) - \frac{1}{2} \frac{\partial}{\partial \phi_n} [I^*] . \quad (B.1)$$

An alternative inertia power modeling matrix $[P_n^*]$ is introduced which is equivalent to $[P_n]$ in the sense that

$$\underline{\dot{\phi}}^T [P_n^*] \underline{\dot{\phi}} = \underline{\dot{\phi}}^T [P_n] \underline{\dot{\phi}} . \quad (B.2)$$

The components of this new matrix have a simplified form expressed in terms of the geometric influence coefficients and the inertia properties of the link.

The component in row ℓ , column m of $[P_n]$ is

$$[P_n]_{\ell; m} = \frac{\partial}{\partial \phi_m} [I^*]_{; n} - \frac{1}{2} \frac{\partial}{\partial \phi_n} [I^*]_{\ell; m} . \quad (B.3)$$

Substituting the expression in Eqn. (2.44) for the scalar components of $[I^*]$, and recognizing that

$$\frac{\partial}{\partial \phi_m} (T_j^T [G_{jk}]_n) = T_j^T \{ [H_{jk}]_{m;n} - \left(\frac{\partial}{\partial \phi_n} T_j \right) T_j^T [G_{jk}]_n \} \quad (B.4)$$

$$\frac{\partial}{\partial \phi_m} T_j = [\tilde{G}_{jk}]_m T_j \quad (B.5)$$

$$T_j T_j^T = [I] \quad (B.6)$$

$$[H_{cj}]_{m;n} = [H_{cj}]_{n;m} . \quad (B.7)$$

We can express the scalar components of $[P_n]$ in the expanded form

$$\begin{aligned} [P_n]_{\ell;m} = & \sum_{j=1}^N \{ M_j \left([H_{cj}]_{m;\ell}^T [G_{cj}]_n \right. \\ & + \frac{1}{2} [H_{cj}]_{m;n}^T [G_{cj}]_{\ell} - \frac{1}{2} [H_{cj}]_{n;\ell}^T [G_{cj}]_m \} \\ & + ([H_{jk}]_{m;\ell} - [\tilde{G}_{jk}]_m [G_{jk}]_{\ell}^T [\Pi_{jk}] [G_{jk}]_n \\ & + [G_{jk}]_{\ell}^T [\Pi_{jk}] ([H_{jk}]_{m;n} - [\tilde{G}_{jk}]_m [G_{jk}]_n) \\ & - \frac{1}{2} ([H_{jk}]_{n;\ell} - [\tilde{G}_{jk}]_n [G_{jk}]_{\ell}^T [\Pi_{jk}] [G_{jk}]_m \\ & - \frac{1}{2} [G_{jk}]_{\ell}^T [\Pi_{jk}] ([H_{jk}]_{n;m} - [\tilde{G}_{jk}]_n [G_{jk}]_m) \} . \end{aligned} \quad (B.8)$$

Since $[P_n]$ is used in a quadratic form, it is possible to substitute the new matrix $[\bar{P}_n]$ defined by

$$[\bar{P}_n] = \frac{1}{2} ([P_n] + [P_n]^T) \quad (B.9)$$

such that

$$\dot{\underline{\Phi}}^T [\bar{P}_n] \dot{\underline{\Phi}} = \dot{\underline{\Phi}}^T [P_n] \dot{\underline{\Phi}} \quad (B.10)$$

The second-order rotational influence coefficients for serial manipulators have the characteristic

$$[H_{jk}]_{m;n} - [H_{jk}]_{n;m} = [\tilde{G}_{jk}]_m [G_{jk}]_n \quad (B.11)$$

This result and Eqn. (B.8) give the general component of $[\bar{P}_n]$ as

$$\begin{aligned} [\bar{P}_n]_{\ell;m} &= \frac{1}{2} ([P_n]_{\ell;m} + [P_n]_{m;\ell}) \quad (B.12) \\ &= \sum_{j=1}^N \{M_j [H_{cj}]_{\ell;m}^T [G_{cj}]_n \\ &\quad + \frac{1}{2} ([H_{jk}]_{\ell;m} + [H_{jk}]_{m;\ell})^T [\Pi_{jk}] [G_{jk}]_n \\ &\quad + \frac{1}{2} [G_{jk}]_m^T [\Pi_{jk}] [\tilde{G}_{jk}]_n [G_{jk}]_{\ell} \\ &\quad + \frac{1}{2} [G_{jk}]_{\ell}^T [\Pi_{jk}] [\tilde{G}_{jk}]_n [G_{jk}]_m \} \end{aligned}$$

Finally, it is convenient to return to a non-symmetric form and define the inertia power modeling matrix $[P_n^*]$ as follows:

$$\begin{aligned}
 [P_n^*]_{\ell;m} = & \sum_{j=1}^N \{ M_j [H_{cj}]_{\ell;m}^T [G_{cj}]_n \\
 & + [H_{jk}]_{\ell;m}^T [\Pi_{jk}] [G_{jk}]_n \\
 & + [G_{jk}]_{\ell}^T [\Pi_{jk}] [\tilde{G}_{jk}]_n [G_{jk}]_m \} .
 \end{aligned} \tag{B.13}$$

The scalar components of the inertia power matrix for flexible manipulators can then be expressed by the equation

$$\begin{aligned}
 [\hat{P}_n^*]_{\ell;m} = & \sum_{j=1}^N \{ M_j [H_{cj}]_{\ell;m}^T [\hat{G}_{cj}]_n \\
 & + [H_{jk}]_{\ell;m}^T [\hat{\Pi}_{jk}] [\hat{G}_{jk}]_n \\
 & + [\hat{G}_{jk}]_{\ell}^T [\hat{\Pi}_{jk}] [\hat{G}_{jk}]_n [\hat{G}_{jk}]_m \} .
 \end{aligned} \tag{B.14}$$

APPENDIX C

VARIATION OF THE INFLUENCE
COEFFICIENTS DUE TO THE MANIPULATOR'S
DEFORMATION

The vector components defining the geometry of flexible serial manipulators will displace from their original line of action due to the deformation of the flexible elements. Consider a vector $\hat{\underline{Q}}$ fixed in link jk which is displaced from its original line of action $\underline{Q}$. Since the displacements due to the manipulator's deformation are small, we can write

$$\hat{\underline{Q}} = \underline{Q} + \Delta \underline{Q} . \quad (C.1)$$

The time rate of change of a vector $\underline{Q}$ fixed in a rigid body jk is defined as

$$\dot{\underline{Q}} = \underline{\omega}_{jk} \times \underline{Q} = ([G_{jk}] \dot{\underline{\Phi}}) \times \underline{Q} . \quad (C.2)$$

Using the mathematical definition of time derivative, Eqn. (C.2) can be rewritten as

$$\frac{\Delta \underline{Q}}{\Delta t} = ([G_{jk}] \frac{\Delta \underline{\phi}}{\Delta t}) \times \underline{Q} \quad (C.3)$$

Canceling Δt from both sides and recognizing that vector $\Delta \underline{\phi}$ portrays the elastic deformation of the arm, $\underline{\beta}$, we can write

$$\Delta \underline{Q} = ([G_{jk}] \underline{\beta}) \times \underline{Q} . \quad (C.4)$$

Substituting Eqn. (C.4) into (C.1) results in

$$\hat{\underline{Q}} = \underline{Q} + ([G_{jk}] \underline{\beta}) \times \underline{Q} . \quad (C.5)$$

The second term on the right hand side of Eqn. (C.5) is negligible since the components of $\underline{\beta}$ are much smaller than those of vector $\underline{Q}$. Thus, Eqn. (C.5) is reduced to

$$\hat{\underline{Q}} \cong \underline{Q} . \quad (C.6)$$

The components of the influence coefficients are defined in terms of the direction cosines of the manipulator's vector components. Therefore, the result of Eqn. (C.6) leads to the conclusion that

$$[\hat{G}_{jk}]_n \cong [G_{jk}]_n \quad (C.7)$$

$$[\hat{G}_p]_n \cong [G_p]_n \quad (C.8)$$

$$[\hat{H}_{jk}]_{m;n} \cong [H_{jk}]_{m;n} \quad (C.9)$$

$$[\hat{H}_P]_{m;n} \cong [H_P]_{m;n} . \quad (C.10)$$

Furthermore, the scalar components of the system inertia matrix $[\hat{I}^*]$ and inertia power matrix $[\hat{P}_n^*]$ are defined in terms of local mass parameters which are invariant with respect to the motion of the arm, and the geometric influence coefficients. Considering Eqns. (C.7) through (C.10), we can conclude that

$$[\hat{I}^*]_{m;n} \cong [I^*]_{m;n} \quad (C.11)$$

$$[\hat{P}_n^*]_{\ell;m} \cong [P_n^*]_{\ell;m} . \quad (C.12)$$

These results indicate that the inertia matrix $[I^*]$ and inertia power matrix $[P_n^*]$ associated with rigid body representation of a manipulator arm can sufficiently describe the inertia properties of the corresponding flexible arm.

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BIOGRAPHICAL SKETCH

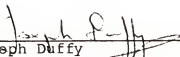
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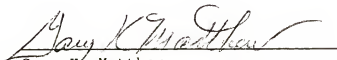
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